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AN H-BASED $\mathbf{A}-\phi$ METHOD WITH A NONMATCHING GRID FOR EDDY CURRENT PROBLEM WITH DISCONTINUOUS COEFFICIENTS *1)

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Abstract

In this paper, we investigate the finite element $\mathbf{A}-\phi$ method to approximate the eddy current equations with discontinuous coefficients in general three-dimensional Lipschitz polyhedral eddy current region. Nonmatching finite element meshes on the interface are considered and optimal error estimates are obtained.

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Key words: Eddy current problem, Finite element $\mathbf{A}-\phi$ method, Nonmatching meshes, Error estimates.

1. Introduction

The eddy current model emerges from Maxwell's equations by formally dropping the displacement currents, which amounts to neglecting capacitive effects (space charges) and provides a reasonable approximation in the low frequency range and in the presence of high conductivity. Various formulations of the eddy current problem have been suggested in [1], which differ in their choice of the primary unknown. Instead of finding magnetic and electric fields directly, the $\mathbf{A}-\phi$ approach is to seek vector and scalar potentials. Although this method increases the number of scalar unknowns and equations, this apparent complication is justified by a better way of dealing with the possible discontinuities in process of the numerical approximations.

It is well-known that the $\mathbf{A}-\phi$ method has been applied to the eddy current model extensively in practice, but further theoretical research in this aspect has rarely shown so far. For some recent relative work, we refer readers to [2, 8-12] for eddy current problem. In [4], Ciarlet and Zou first studied both nodal finite element methods and edge finite element methods for Maxwell equations. Chen et al. in [3] also discussed a fully discrete finite element method for Maxwell equations with discontinuous coefficients by introducing Lagrangian multipliers. In

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this paper, we will study eddy current equations with discontinuous coefficients by using the above methods and give their error estimates in the meanwhile.

This paper is organized as follows. In section 2, the eddy current model in eddy current region is first presented. Second, we give its $\mathbf{A}-\phi$ variational form based on an optimal control formulation of the interface problem and study the feather of its solutions in section 3. Finally, the fully-discrete coupled and decoupled $\mathbf{A}-\phi$ schemes are proposed and their optimal error estimates are obtained in section 4 and 5 respectively.

2. Eddy Current Problem

For simplification, this paper is only concerned with the following eddy current equations in the eddy current region (high conductivity) neglecting the effect of outside source current:

$$\mathbf{curl}\,\mathbf{H} = \sigma\mathbf{E}, \quad \text{in } \Omega\times(0,T), \tag{2.1}$$

$$\mathbf{curl}\,\mathbf{E} = -\frac{\partial(\mu\mathbf{H})}{\partial t}, \quad \text{in } \Omega\times(0,T). \tag{2.2}$$

Here $\Omega\subset\mathbb{R}^3$ is a simply-connected Lipschitz polyhedral domain with connected boundary which is occupied by the dielectric material. We assume that the magnetic permeability parameter μ and the conductivity σ of the medium are discontinuous across an interface $\Gamma\subset\Omega$ respectively, where Γ is the boundary of a simply-connected Lipschitz polyhedral domain Ω_1 with $\overline{\Omega}_1\subset\Omega$ and $\Omega_2=\Omega\setminus\overline{\Omega}_1$. Ω_2 should be multiply-connected as Ω_1 is simply-connected and lies strictly in Ω. Without loss of generality we consider only the case with μ and σ being two piecewise constant function in the domain Ω, namely,

$$\mu=\begin{cases}\mu_1 & \text{in } \Omega_1,\\ \mu_2 & \text{in } \Omega_2,\end{cases} \qquad \sigma=\begin{cases}\sigma_1 & \text{in } \Omega_1,\\ \sigma_2 & \text{in } \Omega_2,\end{cases}$$

and μ_i, σ_i $(i=1,2)$ are positive constants. It is known that magnetic and electric fields must satisfy the following jump conditions across the interface Γ:

$$[\mathbf{H}\times\mathbf{n}] = \mathbf{0}, \tag{2.3}$$

$$[\mathbf{E}\times\mathbf{n}] = \mathbf{0}, \tag{2.4}$$

where $\mathbf{n}$ is the unit outward normal to $\partial\Omega_1$. Throughout the paper, the jump of any function A across the interface Γ is defined as

$$[A] := A_2|_\Gamma - A_1|_\Gamma$$

with $A_i = A|_{\Omega_i}$, $i=1,2$. From (2.1) and (2.4) we can see that,

$$[\frac{1}{\sigma}\,\mathbf{curl}\,\mathbf{H}\times\mathbf{n}] = \mathbf{0}, \quad \text{on } \Gamma\times(0,T). \tag{2.5}$$

We supplement the equation (2.1)-(2.2) with the boundary condition

$$\mathbf{H}\times\mathbf{n} = \mathbf{h}(\mathbf{x},t), \tag{2.6}$$

and the initial condition

$$\mathbf{H}(\mathbf{x},0) = \mathbf{H}_0(\mathbf{x}), \quad \text{in } \Omega, \tag{2.7}$$