Journal of Computational Mathematics, Vol.22, No.6, 2004, 895–904.

A BRANCH AND BOUND ALGORITHM FOR SEPARABLE CONCAVE PROGRAMMING [*1)]

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Abstract

In this paper, we propose a new branch and bound algorithm for the solution of large scale separable concave programming problems. The largest distance bisection (**LDB**) technique is proposed to divide rectangle into sub-rectangles when one problem is branched into two subproblems. It is proved that the **LDB** method is a normal rectangle subdivision(**NRS**). Numerical tests on problems with dimensions from 100 to 10000 show that the proposed branch and bound algorithm is efficient for solving large scale separable concave programming problems, and convergence rate is faster than ω-subdivision method.

Mathematics subject classification: 49M37, 65K05, 90C30.
Key words: Branch and bound algorithm, Separable programming, Largest distance bisection, Normal rectangle subdivision, ω-subdivision.

1. Introduction

The nonconvex programming has come a long way from 60th years of last century (see[6], [9], [4]). It is an important area of mathematical programming, and has many applications in economics, finance, planning, and engineering design. The separable concave programming is a kind of special problems in nonconvex programming. This paper studies the solution of the separable concave programming over a polytope in the form

$$\max\{f(x)|x \in V,\ l \leq x \leq u\}, \tag{\textbf{SCP}}$$

where the objective function has the format

$$f(x) = \sum_{i=1}^{n} f_i(x_i), \tag{1.1}$$

$f_i(x_i), i = 1, 2, \ldots, n$ are convex functions, $x = (x_1, x_2, \ldots, x_n)^T$. The feasible region is the intersection of the polytope V and the rectangle with low bounds $l = (l_1, l_2, \ldots, l_n)^T$ and upper bounds $u = (u_1, u_2, \ldots, u_n)^T$.

Algorithms have been proposed to solve problems (SCP). Falk and Soland(1969) reported a branch and bound algorithm in [1], Thai Quynh Phong et al(1995) proposed a decomposition branch and bound method in [8] to find the global solution of indefinite quadratic programming problems, and tests on problems with 20 concave variables and 200 convex variables show the efficiency of the method. Konno (2001) proposed a branch and bound algorithm to solve large

* Received May 13, 2003; final revised December 11, 2003.
1) This work is supported by National Natural Key Product Foundations of China 10231060.

scale portfolio optimization problems with concave transaction cost (see [2],[4]). The algorithm is based on linear underestimations to objective functions, and is successively used to solve large scale optimal portfolio selection problems with 200 assets and 60 simulated scenarios based on the MAD model (see [2]).

In this paper, we will propose a new branch and bound method to solve the large scale separable concave programming problems. A rectangular subdivision process (largest distance bisection, **LDB**) is proposed. Linear overestimations to objective $f(x)$ are employed to replace $f(x)$, and problems

$$\max\{g^i(x)|x \in V,\ l^i \leq x \leq u^i\} \qquad (LOP)$$

are successively solved, where the objective function $g^i(x) = \sum_{j=1}^n g_j^i(x_j)$, and $g_j^i(x_j),\ j = 1, 2, \dots, n$, are linear overestimations to functions $f_j(x_j)$ over the set $S_i = [l^i, u^i]$.

Rectangle subdivision processes play an important role in branch and bound methods. As will be seen later, the concept of "normal rectangular subdivision" introduced in reference (3) will be concerned in the method proposed in this paper. The class of normal subdivision methods includes the exhaustive bisection, ω-subdivision and adaptive bisection (see [8]). A new bisection technique will be proposed and it will be shown that the proposed bisection method belongs to the class of normal rectangular subdivisions. Experiments on quadratic functions with different dimensions show that the iterative process with the proposed bisection technique can converge effectively. The quadratic function is a type concave function when we set its second order coefficients negative. The dimensions of tested problems ranges from 100 to 10000, large scale problems. The coefficients of these tested problems are randomly generated from uniform distribution. Tested functions are very important in economical and financial field, and usually used to express utilities or costs (see [5]). These functions often exhibit concave characteristics when they are used to denote cost functions or utility functions under the rule of margin cost (utility) decrease. When net returns or expected utilities are maximized, the resulting problem usually is a separable concave programming (see [7],[5],[2],).

A series of numerical experiments is presented and shows the efficiency of the proposed bisection method. It also shows that the algorithm can solve problems of practical size in an efficient way.

The rest of the paper is organized as follows. In Section 2, we will describe the new branch and bound algorithm. In section 3, we discuss the construction of a normal rectangular subdivision, and the largest distance bisection strategy will also be presented in this section. In section 4, we conduct a series of numerical tests, and present comparisons with some different bisection methods. Conclusions are given in section 5.

2. A Branch and Bound Algorithm

In this section, we describe the branch and bound method which bases upon a chosen normal rectangular subdivision process.

Let $S_0 = \{l_i \leq x_i \leq u_i,\ i = 1, 2, \dots, n\}$ be a rectangle. We replace the convex functions $f_i(x_i)$ in $f(x)$ by an overestimated linear function $g_i^0(x_i)$ over S_0(see Figure 1),

$$g_i^0(x_i) = \delta_i x_i + \eta_i, \quad i = 1, 2, \dots, n \tag{2.1}$$

where

$$\delta_i = \frac{f_i(u_i) - f_i(l_i)}{u_i - l_i}, \quad \eta_i = f_i(l_i) - \delta_i l_i, \quad i = 1, 2, \dots, n. \tag{2.2}$$

Let

$$g^0(x) = \sum_{i=1}^n g_i^0(x_i),$$

then $g^0(x)$ is the convex envelope of the function $f(x)$ over the set S_0. We solve the linear