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SEQUENTIAL CONVEX PROGRAMMING METHODS FOR SOLVING LARGE TOPOLOGY OPTIMIZATION PROBLEMS: IMPLEMENTATION AND COMPUTATIONAL RESULTS *1)

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Abstract

In this paper, we describe a method to solve large-scale structural optimization problems by sequential convex programming (SCP). A predictor-corrector interior point method is applied to solve the strictly convex subproblems. The SCP algorithm and the topology optimization approach are introduced. Especially, different strategies to solve certain linear systems of equations are analyzed. Numerical results are presented to show the efficiency of the proposed method for solving topology optimization problems and to compare different variants.

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Key words: Large scale optimization, Topology optimization, Sequential convex programming method, Predictor-corrector interior point method, Method of moving asymptotes.

1. Introduction

The method of moving asymptotes (MMA) was introduced by Svanberg [7] in 1987. To prove global convergence and to stabilize the algorithm, Zillober [8] added a line search procedure and called it the sequential convex programming (SCP) method. Both methods are proved to be efficient tools in the context of mechanical structural optimization, see for instance the comparative study of Schittkowski et al. [3], especially since displacement dependent constraints are approximated very well. But also optimization problems from other areas can be solved very efficiently in certain situations [5]. In a recent paper of Zillober et al. [12], it is shown how very large scale optimal control problems with partial elliptic equations can be solved after a full discretization.

Zillober [9] extended the approach to a generally applicable mathematical programming framework, and in [10] the predictor-corrector interior point method for solving the convex nonlinear subproblems was introduced. Moreover, a Fortran-code with name SCPIP [11] was developed which is in practical use in many academic and commercial applications.

The main focus of this paper is to show how the SCP method can be applied to solve large-scale topology optimization problems. These problems can become extremely large and possess dense Hessians of the objective function. The mathematical structure is easily analyzed and a large number of scalable test problems is obtained in a straightforward way.

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To describe the SCP method, we consider the general nonlinear programming problem

$$\begin{array}{lll} \min & f(x), & x \in \mathbb{R}^n, \\ \text{s.t.} & h_j(x) = 0, & j = 1, ..., m_{eq}, \\ & h_j(x) \leq 0, & j = m_{eq}+1, ..., m, \\ & \underline{x_i} \leq x_i \leq \overline{x_i}, & i = 1, ..., n. \end{array} \tag{1.1}$$

The functions f and h_j, $j = 1, \ldots, m$, are defined on $X := \{x \,|\, \underline{x}_i \leq x_i \leq \overline{x}_i,\ i = 1, \ldots, n\}$, are assumed to be continuous in X and at least twice continuously differentiable in the interior of X. The feasible region is assumed to be non-empty.

The objective function of (1.1) is approximated by a uniformly convex function, inequality constraints by convex functions, and equality constraints by linear functions. Thus, (1.1) is replaced by a separable, convex, and nonlinear subproblem which is much easier to solve. Numerical results show the advantages of an interior point method for solving the subproblem. It is possible to reduce the size of the internally generated linear systems, where the major part of the computing is spent, to m, which is favorable when m is small compared to n as is the case for topology optimization problems. Another possibility is to reduce the size of linear subsystems to n. A small number of variables and a large number of constraints is a typical situation for many sizing problems in structural optimization. Moreover, there is a third possibility to formulate linear systems with $n + m$ equations and variables, by which special sparsity patterns can be exploited. The first two approaches will be compared by numerical tests.

The outline of the paper is as follows. In Section 2 the SCP method is formulated and the SCPIP code is briefly introduced. Topology optimization, our main source for generating test problems, is described in Section 3. Section 4 contains numerical results.

2. The Sequential Convex Programming Method

Similar to most other nonlinear programming algorithms, the SCP method replaces problem (1.1) in the k-th step by a subproblem

$$\begin{array}{lll} \min & f^k(x), & x \in \mathbb{R}^n, \\ \text{s.t.} & h_j^k(x) = 0, & j = 1, ..., m_{eq}, \\ & h_j^k(x) \leq 0, & j = m_{eq}+1, ..., m, \\ & \underline{x_i}' \leq x_i \leq \overline{x_i}', & i = 1, ..., n. \end{array} \tag{2.1}$$

If we define

$$\phi_i(g, z, y, x) = \frac{\partial g(y)}{\partial x_i} \left(\frac{(z_i - y_i)^2}{z_i - x_i} - (z_i - y_i) \right), \tag{2.2}$$

where $g : \mathbb{R}^n \longrightarrow \mathbb{R}$, $x, y, z \in \mathbb{R}^n$ with $x = (x_1, \ldots, x_n)^T$, $y = (y_1, \ldots, y_n)^T$, and $z = (z_1, \ldots, z_n)^T$, the approximation of the objective function f^k in the $k-$th step is defined by

$$f^k(x) = f(x^k) + \sum_{i \in I_+^k} \left(\phi_i(f, U^k, x^k, x) + \tau_i^k \frac{(x_i - x_i^k)^2}{U_i^k - x_i} \right) + \sum_{i \in I_-^k} \left(\phi_i(f, L^k, x^k, x) + \tau_i^k \frac{(x_i - x_i^k)^2}{x_i - L_i^k} \right),$$

where $I_+^k = \{i : \frac{\partial f(x^k)}{\partial x_i} \geq 0\},\ I_-^k = \{1, \ldots, n\} \setminus I_+^k$. Inequality constraints are approximated by