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THE MORTAR ELEMENT METHOD FOR A NONLINEAR BIHARMONIC EQUATION *1)

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Abstract

The mortar element method is a new domain decomposition method(DDM) with nonoverlapping subdomains. It can handle the situation where the mesh on different subdomains need not align across interfaces, and the matching of discretizations on adjacent subdomains is only enforced weakly. But until now there has been very little work for nonlinear PDEs. In this paper, we will present a mortar-type Morley element method for a nonlinear biharmonic equation which is related to the well-known Navier-Stokes equation. Optimal energy and H^1-norm estimates are obtained under a reasonable elliptic regularity assumption.

Mathematics subject classification: 65F10, 65N30, 65N55.
Key words: Mortar method, Nonlinear biharmonic equation, H^1-norm error, Energy norm error.

1. Introduction

In recent years, the mortar finite element method as a special domain decomposition methodology appears very attractive because it can handle the situation where meshes on different subdomains need not align across interfaces, and the matching of the solutions on adjacent subdomains is only enforced weakly. We refer to[3],[5],[6]for the general presentation of the mortar element method. Recently, there have been many works in constructing efficient iterative solvers for the discrete system resulting from the mortar element method (cf. In [1],[2],[20],[17],[21],[22]). So far, many mortar element methods were presented for solving linear elliptic problems. Very little work has been done for the nonlinear problems. In this direction, a mortar finite element for quasilinear elliptic problems was considered in [14], while the mortar element methods for some variational inequalities were developed in [4], [12].

The mortar element method for biharmonic problems also attracted many authors' attentions. For instance, the mortar finite element method for some plate elements, like the conforming Hsieh-Clough-Tocher, the reduced Hsieh-Clough-Tocher and a nonconforming Morley element, was studied by Marcinkowski in [15]. But his error estimate requires that the solution is very smooth (in $H^4(\Omega)\cap H_0^2(\Omega)$) which is generally not valid, even for some convex polygonal domains. Recently, Huang, Li and Chen [13] extended this work and obtained an optimal error estimate with a weaker elliptic regularity assumption ($H^3(\Omega)\cap H_0^2(\Omega)$). An efficient multigrid for such kind of mortar element method was proposed in [23]. But till now there have been no results for the nonlinear counterparts. In this paper, we shall design an effective mortar element method for a nonlinear biharmonic equation which is related to the well known Navier-Stokes equation. Optimal energy and H^1-norm estimates are obtained under the weaker elliptic regularity assumption($H^3(\Omega)\cap H_0^2(\Omega)$).

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This paper is organized as follows. Section 2 introduces the model problem. In section 3, we shall present the mortar-type Morley element method, some prelimilary shall be given in this section. Optimal energy amd H_1 norm error estimates shall be studied in section 4.

2. Model Problem

We consider the following nonlinear biharmonic equation:

$$\begin{cases} \frac{1}{R_e}\triangle^2 u = Bu + f & \text{in } \Omega, \\ u = \partial_n u = 0 & \text{on } \partial\Omega, \end{cases} \tag{2.1}$$

where Ω is a convex polygonal domain in R^2, $n = (n_1, n_2)$ denotes the unit outward normal vector along the boundary $\partial\Omega$, and

$$Bu = \partial_x(\partial_y u \triangle u) - \partial_y(\partial_x u \triangle u) = \partial_y u \triangle \partial_x u - \partial_x u \triangle \partial_y u.$$

Let $H^r(\Omega)$ denote the standard Sobolev space of order $r \geq 0$ with respect to domain Ω, equipped with the standard norm $\|\cdot\|_r$. Define the subspace

$$H_0^2(\Omega) = \{v \in H^2(\Omega) : v = \partial_n v = 0 \text{ on } \partial\Omega\}.$$

Let $|\cdot|_r$ be the seminorm over the Sobolev space $H^r(\Omega)$. It is known that $|\cdot|_2$ is a norm over the space $H_0^2(\Omega)$ and (cf. [8] for details)

$$|v|_2 = \|\triangle v\|_0, \quad \forall v \in H_0^2(\Omega).$$

The variational form of (2.1) is to find $u \in H_0^2(\Omega)$ such that

$$\frac{1}{R_e} a(u, v) = (\partial_x u \triangle u, \partial_y v) - (\partial_y u \triangle u, \partial_x v) + (f, v), \quad \forall v \in H_0^2(\Omega), \tag{2.2}$$

where f is a function in $L^2(\Omega)$, and

$$a(u, v) = \int_\Omega \triangle u \triangle v dx dy,$$

$$(f, v) = \int_\Omega f v dx dy.$$

By the Sobolev embedding Theorem, we know that

$$\|\nabla v\|_{L^4} \leq C_0 |v|_2, \quad \text{and} \quad \|v\|_0 \leq C_1 |v|_2, \quad \forall v \in H_0^2(\Omega). \tag{2.3}$$

Here $\|\cdot\|_{L^4}$ is the norm over the space $L^4(\Omega)$. In this paper C with or without subscript and supscript denotes a positive constant.

It is known ([7],[10]) that (2.2) has a unique solution $u \in H_0^2(\Omega)$ which satisfies

$$\|\triangle u\|_0 \leq C_1 R_e \|f\|_0$$

under the assumption

$$R_e < \sqrt{\frac{1}{C_0^2 C_1 \|f\|_0}}. \tag{2.4}$$