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A NEW FILLED FUNCTION METHOD FOR INTEGER PROGRAMMING *1)

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Abstract

The Filled Function Method is a class of effective algorithms for continuous global optimization. In this paper, a new filled function method is introduced and used to solve integer programming. Firstly, some basic definitions of discrete optimization are given. Then an algorithm and the implementation of this algorithm on several test problems are showed. The computational results show the algorithm is effective.

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1. Introduction

An integer programming problem is very difficult to be solved. In [1], it is pointed out that the integer programming with a linear objective function and quadratic constrained is algorithmically unsolved, that is, there no algorithms can be created to solve them. Because of these reasons, the approximate algorithms have been rapidly developed in recent years(see [2]). In publishing literatures, approximate algorithms for integer programming can be sorted into two categories: stochastic approach(see [3-5]) and deterministic approach(see [6-8]). The filled function method applied for continuous global optimization is proposed firstly by Ge in [9]. This method is consist of two stages: 1. finding a local minimizer, x_1^*, of the original continuous global optimization by any local minimization method; 2. constructing a filled function and then minimizing this filled function to get another local minimizer of original problem whose objective function value is smaller than that of x_1^*. Repeat the above two steps to find the global minimizer of original problem. The key of this method is to construct a filled function. Now we extend the filled function method to solve the integer programming problem. In this paper, a new filled function is introduced. At the same time, we show a new filled function method for discrete global optimization by this new filled function. This is an approximate and direct approach. By solving some testing problems, it is showed to be an effective and efficient method. In [10], Tian and Zhang give a filled function with two parameters. But the filled function in this paper has only one parameter. Moreover this parameter is chosen easily.

We consider integer programming problem given by **(P)**

$$\begin{cases} min \ \ f(x) \\ s.t. \ \ x \in \Omega \cap R_I^n \end{cases}$$

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where Ω is a bounded closed box with all vertices integral, R_I^n is the set of integer points in R^n. Furthermore, we suppose that $f(x)$ is coercive, that is, $f(x) \to +\infty$ as $\|x\| \to +\infty$. So all local minimizers of $f(x)$ which values are more small can't be on the boundary of $\Omega \cap R_I^n$ if Ω is sufficiently large. In the following of this paper, we let

$$X_I = \Omega \cap R_I^n.$$

The paper is organized as follows: Section 2 gives the preliminary knowledge about discrete optimization; A new filled function for discrete optimization is proposed in section 3; We show an algorithm and the numerical experiments in section 4; Some conclusions are in section 5.

2. Preliminaries

In this section, we firstly introduce some basic definitions and method for discrete optimization.

Definition 2.1 *The set of all directions in discrete analysis is defined by*

$$D = \{e_i, -e_i; i = 1, 2, ..., n\}.$$

where e_i is the $i-th$ unit vector which is the $n-dimensional$ vector with the $i-th$ component equal to one and all other components equal to zero.

Definition 2.2 *For any $x \in R_I^n$, the neighborhood of x is defined as*

$$N(x) = \{x, x \pm e_i : i = 1, 2, ..., n\}.$$

Definition 2.3 *An integer point $x_1^* \in X_I$ is called a discrete local minimizer of $f(x)$ if $f(x) \geq f(x_1^*)$, for all $x \in N(x_1^*) \cap X_I$. Furthermore, if $f(x) > f(x_1^*)$ for all $x \in (N(x_1^*) \cap X_I)\backslash x_1^*$, then x_1^* is called a strict discrete local minimizer of $f(x)$.*

Suppose x_1^* is a (strict) discrete local minimizer of $f(x)$, then x_1^* is a (strict) discrete local maximizer of $-f(x)$.

Definition 2.4 *An integer point $x^* \in X_I$ is called a discrete global minimizer of $f(x)$ if $f(x) \geq f(x^*)$, for all $x \in X_I$. Furthermore, if $f(x) > f(x^*)$ for all $x \in X_I\backslash x^*$, then x^* is called a strict discrete global minimizer of $f(x)$.*

Suppose x^* is a (strict) discrete global minimizer of $f(x)$, then x^* is a (strict) discrete global maximizer of $-f(x)$.

Definition 2.5 *Suppose x_1^* is a discrete local minimizer of $f(x)$. A function $P(x, x_1^*)$ is said to be a discrete filled function of $f(x)$ at x_1^* if $P(x, x_1^*)$ satisfies the following properties:*

1. *x_1^* is a strict discrete local maximizer of $P(x, x_1^*)$;*
2. *if $f(x) \geq f(x_1^*)$ and $f(x + d_i) \geq f(x_1^*), \forall d_i \in D$, $x \neq x_1^*$, then x is not a local minimizer of $P(x, x_1^*)$;*
3. *for x_1, x_2, if $\|x_1 - x_1^*\| > \|x_2 - x_1^*\| > 0$ and $f(x_1)$, $f(x_2) \geq f(x_1^*)$, then $P(x_1, x_1^*) < P(x_2, x_1^*)$;*
4. *for x_1, x_2, if $\|x_1 - x_1^*\| > \|x_2 - x_1^*\| > 0$ and $f(x_2) \geq f(x_1^*) > f(x_1)$, then $P(x_1, x_1^*) > P(x_2, x_1^*)$.*

Remark. The 2-th item of this definition notes: if there exists $d_{i_0} \in D$ such that $f(x + d_{i_0}) < f(x_1^*)$, then let $x_0 = x + d_{i_0}$ to find another discrete local minimizer x_2^* of $f(x)$ which holds $f(x_2^*) < f(x_1^*)$.