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CONVERGENCE ANALYSIS OF MORLEY ELEMENT ON ANISOTROPIC MESHES *1)

Shi-peng Mao
(*ICMSEC, Academy of Mathematics and System Sciences, Chinese Academy of Sciences, Beijing 100080, China*)

Shao-chun Chen
(*Department of Mathematics, Zhengzhou University, Zhengzhou 450052, China*)

Abstract

The main aim of this paper is to study the convergence of a nonconforming triangular plate element-Morley element under anisotropic meshes. By a novel approach, an explicit bound for the interpolation error is derived for arbitrary triangular meshes (which even need not satisfy the maximal angle condition and the coordinate system condition), the optimal consistency error is obtained for a family of anisotropically graded finite element meshes.

Mathematics subject classification: 65N30, 65N15.
Key words: Anisotropic meshes, Interpolation error, Consistency error, Morley element.

1. Introduction

It is well-known that regular assumption or quasi-uniform assumption[9,12] of finite element meshes is a basic condition in the convergence analysis of finite element approximation both for conventional conforming and nonconforming elements. However, with the development of the finite element methods and its applications to more fields and more complex problems, the above conventional meshes conditions become a severe restriction for the finite element methods. For example, the solution may have anisotropic behavior in parts of the domain. This means that the solution varies significantly only in certain directions. In such cases, it is an obvious idea to reflect this anisotropy in the discretization by using anisotropic meshes with a small mesh size in the direction of the rapid variation of the solution and a larger mesh size in the perpendicular direction.

Indeed, some early papers have been written to prove error estimates under more general conditions (refer to [7, 15]). Recently, much attention is paid to FEMs under anisotropic meshes. In particular, for second order problems and rectangular meshes, we refer to Acosta [1,2], Apel[3−6], Chen[10,11], Duran[13,14], Shenk[22] and references therein. Above all, it is now well known that the regularity assumption is not needed. As to fourth order problems, the plate bending problem for example, only some rectangular elements have been concerned, interested reader can refer to [11] for Adini's element and [19] for bicubic Hermite element. However, up to now, there are no papers on anisotropic triangular plate elements, especially for nonconforming ones. This paper is devoted to fill the gap of it.

It is known that the nonconforming Morley element is an effective element for the plate bending problem. This quadratic triangular element is particularly attractive, because of its simple structure and low degrees of freedom. However, since the continuity of Morley element is very weak (nonconforming non-C^0 element), even under quasi-uniform meshes, the error

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estimate of it is not easy and has been explored a long way (refer to [17, 20, 6, 21]). In this paper, we consider the plate bending problem discretized with the nonconforming Morley element under anisotropic triangular meshes. Since the technique developed to estimate the local interpolation error (refer to [4, 10]) is not convenient to be applied for triangular elements, we turn to other tricks. By using of the special properties of the shape function space of Morley element and the results of Poincaré inequality (refer to [8, 18]), we derive an explicit bound of its interpolation error under arbitrary triangular meshes. The consistency error is even more hard to be treated. In order to obtain the optimal consistency error, we have to consider a special type of product anisotropic triangular meshes, namely, tensor product meshes. As to more general anisotropic triangular meshes, we are still work on them.

The outline of the paper is as follows. In the next section, after introducing the nonconforming Morley element approximation to the plate bending problem, we derive the interpolation error of it under arbitrary triangular meshes. In section 3, the optimal anisotropic consistency error of Morley element is obtained by a novel approach under a family of anisotropically graded finite element meshes . In order to verify the validity of theoretical analysis, some numerical experiments are carried out in section 4.

2. The Interpolation Error Estimate on Arbitrary Triangular Meshes

We consider the plate bending problem[12]:

$$\begin{cases} \triangle^2 u = f, & \text{in } \Omega, \\ u = \dfrac{\partial u}{\partial n} = 0, & \text{on } \partial\Omega, \end{cases} \tag{2.1}$$

where Ω denotes a plane polygonal domain, $f \in L^2(\Omega)$ is the applied force, n is the unit outward normal along the boundary $\partial\Omega$. The related variational form is :

$$\begin{cases} \text{Find } \ u \in H_0^2(\Omega), \ \text{ such that} \\ a(u,v) = (f,v), \quad \forall v \in H_0^2(\Omega), \end{cases} \tag{2.2}$$

where

$$\begin{aligned} &a(u,v) = \int_\Omega A(u,v)dxdy, \\ &A(u,v) = \triangle u \triangle v + (1-\sigma)(2u_{xy}v_{xy} - u_{xx}v_{yy} - u_{yy}v_{xx}), \\ &(f,v) = \int_\Omega fvdxdy, \\ &H_0^2(\Omega) = \{v \in H^2(\Omega), v = \frac{\partial v}{\partial n} = 0, \text{ on } \partial\Omega\} \end{aligned}$$

and σ is the Poisson ratio, $0 < \sigma < \frac{1}{2}$, $u_{xy} = \frac{\partial^2 u}{\partial x \partial y}$, etc.

Clearly, the above bilinear form $a(\cdot,\cdot)$ is bounded and coercive :

$$\begin{cases} |a(v,w)| \le (1+\sigma)|v|_{2,\Omega}|w|_{2,\Omega}, \quad v,w \in H_0^2(\Omega) \\ a(v,v) \ge (1-\sigma)|v|_{2,\Omega}^2, \quad v \in H_0^2(\Omega). \end{cases} \tag{2.3}$$

Throughout this paper, we adopt the standard conventions for Sobolev norms and seminorms of a function v defined on an open set G:

$$\|v\|_{m,G} = \left(\int_G \sum_{|\alpha| \le m} |D^\alpha v|^2 \right)^{\frac{1}{2}},$$