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DOMAIN DECOMPOSITION WITH NONMATCHING GRIDS FOR EXTERIOR TRANSMISSION PROBLEMS VIA FEM AND DTN MAPPING *1)

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Dedicated to the 70th birthday of Professor Lin Qun

Abstract

In this paper, we are concerned with a non-overlapping domain decomposition method (DDM) for exterior transmission problems in the plane. Based on the natural boundary integral operator, we combine the DDM with a Dirichlet-to-Neumann (DtN) mapping and provide the numerical analysis with nonmatching grids. The weak continuity of the approximation solutions on the interface is imposed by a dual basis multiplier. We show that this multiplier space can generate optimal error estimate and obtain the corresponding rate of convergence. Finally, several numerical examples confirm the theoretical results.

Mathematics subject classification: 65N30, 65N55.
Key words: Domain decomposition, Nature boundary element, Nonmatching grids, Weak continuity, D-N alternating, Dual basis, Projection operator, Error estimate.

1. Introduction

Domain decomposition method (DDM) with nonmatching grids is a kind of nonconforming finite element methods. In the past few years, there is a fast growing interest in this field (see [1], [2], [5], [7]). This kind of DDM allows different discretizations in different nonoverlapping subdomains by some Lagrange multiplier. This nonconforming element method also allows for local refinement in only certain subregions of the computational domain. Hence, it is suitable for parallel computing (see [6]).

The key point to deal with the nonmatching grids is how to choose the matching condition so that the resulting approximation problem possesses the optimal error estimate. The approximate solutions must satisfy some weak continuity such as the integration matching condition, whereas the pointwise matching.

In this paper, we propose a new class of multiplier space for the exterior unbounded problems with annular interfaces, which is based on the idea of dual basis multiplier (refer to [7]). We impose weak continuity conditions in the sense that the jump of the DDM solution across the interface is required to be orthogonal to a space of test functions. Due to the character of annular interface that there is no intersections between any of two subregions, it is easier for us to construct efficient and practical multiplier. The basis functions of the multiplier spaces

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are generated by a set of simple functions with local compact supports. The resulting discrete system is still symmetric and positive definite. It will be shown such construction guarantee the optimal energy error estimate for the approximation solutions and the discrete formulation is easy to be solved.

The outline of the paper is as follows. In Section 2, we present the coupled variational formulation for the exterior transmission problem by the finite element method and the natural boundary element method (FEM-NBEM). Then we make a finite element discretization with nonmatching grids for this coupled system in Section 3, and the construction of the multiplier spaces is also introduced. It will be shown that the nonconforming approximation posseses the optimal energy error estimate. In Section 4, we give a D-N alternating method to solve the discrete system and show that this D-N algorithm is convergent and independent of the finite element meshes. Finally, in Section 5, we illustrate these theoretical results by using some numerical examples.

2. FEM-NBEM Coupling

As a model problem, we consider a second order elliptic equation in divergence form coupled with the Laplace equation in the exterior unbounded region. Let Ω_0 be a bounded domain of $\mathbb{R}^2$ with a Lipschitz-continuous boundary Γ_0. Ω_1 is the annular region bounded by Γ_0 and another smooth closed curve Γ_1 that is strictly contained in $\mathbb{R}^2\backslash\bar{\Omega}_0$ (see Figure 1). We denote by Ω_c the complement of $\bar{\Omega}_0\bigcup\bar{\Omega}_1$. Assume that $g \in H^{1/2}(\Gamma_0)$ and $f \in L^2(\Omega_1)$, then the exterior transmission problem reads as: find u such that

$$u_1 = g, \quad \text{on} \quad \Gamma_0 \quad -\operatorname{div}(A\nabla u_1) = f \quad \text{in} \quad \Omega_1 \tag{2.1a}$$

$$u_1 = u_c \quad \text{and} \quad (A\nabla u_1)\cdot \boldsymbol{n} = \frac{\partial u_c}{\partial \boldsymbol{n}} \quad \text{on} \quad \Gamma_1 \tag{2.1b}$$

$$-\Delta u_c = 0, \quad \text{in} \quad \Omega_c \quad u_c(x) = O(1) \quad \text{as} \quad |x| \to \infty \tag{2.1c}$$

where $\boldsymbol{n} = (n_1, n_2)^T$ denotes the unit outward normal to Γ_1 and A is uniformly symmetric pos-

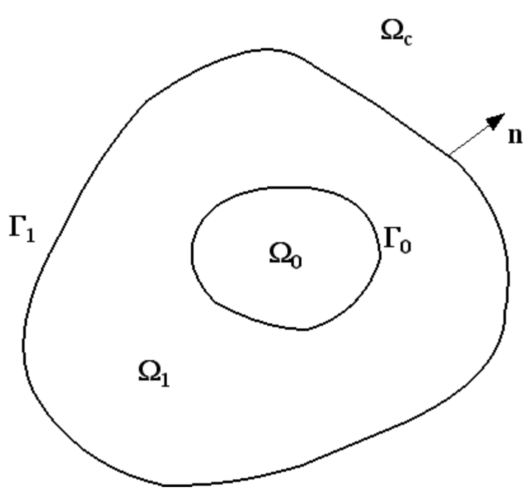

Figure 1: The domain of transmission problem

itive definite matrix with Lipschitz-continuous coefficients, that is to say, there exists constants