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A NEW SQP-FILTER METHOD FOR SOLVING NONLINEAR PROGRAMMING PROBLEMS *1)

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Abstract

In [4], Fletcher and Leyffer present a new method that solves nonlinear programming problems without a penalty function by SQP-Filter algorithm. It has attracted much attention due to its good numerical results. In this paper we propose a new SQP-Filter method which can overcome Maratos effect more effectively. We give stricter acceptant criteria when the iterative points are far from the optimal points and looser ones vice-versa. About this new method, the proof of global convergence is also presented under standard assumptions. Numerical results show that our method is efficient.

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Key words: Nonlinear programming, Sequential quadratic programming, Filter, Restoration phase, Maratos affects, Global convergence, Multi-objective optimization, Quadratic programming subproblem.

1. Introduction

This paper is concerned with a new SQP-Filter method which not only has good convergence properties but also shows encouraging numerical results. We consider the general nonlinear programming problem(NLP)

$$\begin{array}{ll} min & \qquad f(x) \\ s.t. & c_i(x)=0, i=1,\ldots,m_e, \\ & c_i(x)\geq 0, i=m_e+1,\ldots,m, \end{array} \tag{1.1}$$

where f and $c_i (i=1,2,\cdots,m)$ are twice continuously differentiable functions. Let $c(x)=(c_1(x),c_2(x),\cdots,c_m(x))^T$, $c_{\mathcal{E}}=(c_i)_{i\in\mathcal{E}}$ and $c_{\mathcal{I}}=(c_i)_{i\in\mathcal{I}}$, where $\mathcal{E}=\{1,\ldots,m_e\}$ and $\mathcal{I}=\{m_e+1,\ldots,m\}$ denote the indices of equality and inequality constraints, respectively. With these notations the Lagrangian function associated with (1.1) is given by

$$\mathbb{L}(x,\lambda)=f(x)-\lambda^T c(x), \tag{1.2}$$

where $\lambda=(\lambda_{\mathcal{E}}^T,\,\lambda_{\mathcal{I}}^T)\in\Re^m$, $\lambda_{\mathcal{I}}\geq 0$. We denote the gradients of f by $g=g(x)=\nabla f(x)$, the Jacobian of the constraints by $A(x)=\nabla c(x)^T$, $A_{\mathcal{E}}=A_{\mathcal{E}}(x)=\nabla c_{\mathcal{E}}^T$ and $A_{\mathcal{I}}=A_{\mathcal{I}}(x)=\nabla c_{\mathcal{I}}^T$ corresponding to the equality and inequality constraints respectively. Superscript $^{(k)}$ refers to iterations indices and $f^{(k)}$ is taken to mean $f(x^{(k)})$ etc. Quantities relating to a local solutions x^* of (1.1) are superscripted with a $*$.

In the pioneering work[4], Fletcher and Leyffer present the filter technique for the globalization of SQP methods, which avoids the use of multipliers or penalty parameters. Instead, the

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acceptability of steps is determined by comparing the constraint violation and objective function value. The new iterative point is acceptable to the filter if either feasibility or objective function value is sufficiently improved in comparison to all iterative points bookmarked in the current filter. The promising numerical results in[4] led to a growing interest in filter methods in recent years. By strengthening the acceptant criteria of the filter, the global convergence of three variants of trust-region SQP-Filter methods is shown in Fletcher and Leyffer [4].

In this paper we propose a new SQP-Filter method that ensures global convergence. The original SQP-Filter methods in [1, 5, 6] can be affected by the well known Marotos effect. In fact, these methods use constraint violation and objective value in the filter. By the Marotos effect a full SQP-step can lead to an increase of both these filter components even arbitrarily close to a regular minimizer(see [15], P.539). This makes the full SQP-step unacceptable for the filter and can prohibit fast local convergence. To avoid this, we use different filters and different acceptant criteria by the measurement of constraint violation. And two obvious advantages are that our method converges quickly when the points are far from the optimal points and our method can prevent Marotos effect efficiently when the iterative points come closer to the optimal points.

The paper is organized as follows. In section 2 we describe briefly the trust-region SQP-Filter method of Fletcher, Leyffer and Toint[4]. In section 3 we present the main idea of our new SQP-Filter method. In section 4 we give out the SQP-Filter algorithm and its convergence analysis. Finally at section 5 we give some numerical results which show that our method is very efficient.

2. Fletcher's Trust-region SQP-Filter Method

In this section we recall the trust-region SQP-Filter method proposed by Fletcher and Leyffer[4] for (1.1).

Given the current iterative point $x^{(k)}$ and a trust-region radius $\rho^{(k)}$, the computation of the step is based on the approximate solution of the QP-subproblems $QP_{FL}(x^{(k)}, \rho^{(k)})$

$$\begin{array}{ll} min & \hat{\Psi}^{(k)}(d) = \frac{1}{2}d^T W^{(k)} d + d^T g^{(k)} \\ s.t. & c_i^{(k)} + d^T a_i^{(k)} = 0, i = 1, \ldots, m_e, \\ & c_i^{(k)} + d^T a_i^{(k)} \geq 0, i = m_e + 1, \ldots, m, \\ & \|d\|_\infty \leq \rho^{(k)}, \end{array} \tag{2.1}$$

where $W^{(k)} = \nabla^2 \mathbb{L}(x^{(k)}, \lambda)$ denotes the exact Hesse matrix of Lagrangian function at point $x^{(k)}$. For convenience of description, Fletcher and Leyffer in [4] give the concepts of constraint violation, dominance and filter as follows

Definition 2.1. *For the nonlinear programming problem (*1.1*), we define its constraint violation function $h(c(x))$ by*

$$h(c(x)) = \sum_{i \in \mathcal{E}} |c_i(x)| + \sum_{i \in \mathcal{I}} max\{-c_i(x), 0\}. \tag{2.2}$$

For simplicity, we use $h^{(k)}$ and $f^{(k)}$ to denote values of $h(c(x))$ and $f(x)$ evaluated at point $x^{(k)}$, respectively.

Definition 2.2. *A pair $(h^{(k)}, f^{(k)})$ is said to dominate another pair $(h^{(l)}, f^{(l)})$ if and only if both $h^{(k)} \leq h^{(l)}$ and $f^{(k)} \leq f^{(l)}$.*

With this concept Fletcher and Leyffer give the definition of a filter, which will be used in a trust-region type algorithm as criteris for accepting or rejecting a trial step.