Journal of Computational Mathematics, Vol. 24, No. 6, 2006, 761–770.

AN ADAPTIVE NONMONOTONIC TRUST REGION METHOD WITH CURVILINEAR SEARCHES *1)

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Abstract

In this paper, an algorithm for unconstrained optimization that employs both trust region techniques and curvilinear searches is proposed. At every iteration, we solve the trust region subproblem whose radius is generated adaptively only once. Nonmonotonic backtracking curvilinear searches are performed when the solution of the subproblem is unacceptable. The global convergence and fast local convergence rate of the proposed algorithms are established under some reasonable conditions. The results of numerical experiments are reported to show the effectiveness of the proposed algorithms.

Mathematics subject classification: 65K05, 90C30.
Key words: Unconstrained optimization, Preconditioned gradient path, Trust region method, Curvilinear search.

1. Introduction

Consider the following unconstrained nonlinear programming problem

$$\min_{x\in R^n} f(x), \tag{1.1}$$

where $f(x)$ is a real-valued continuously differentiable function.

Trust region methods are very popular for solving problem (1.1). Many different versions have been suggested by using trust region strategy. The idea of combining the optimal path and the modified gradient path with the trust region strategy to solve (1.1) is originally due to Bulteau and Vial [2]. The two paths can be expressed by eigenvalues and eigenvectors of the Hessian matrix of the quadratic model function. However, the calculation of the full eigensystem of a symmetric matrix is usually time-consuming. To overcome this difficulty, Xu and Zhang in [14] have employed the stable Bunch-Parlett factorization method to factorize the Hessian to form a preconditioned optimal path within the trust region for unconstrained optimization. This idea is extended to the modified gradient path [7] and we call the corresponding path the preconditioned modified gradient path.

In addition, Nocedal and Yuan [9] suggest a combination of the trust region and line search methods, and the new algorithm preserves the strong convergence properties of trust region methods. This algorithm is extended to the nonmonotone case in [16]. Recently, Zhu [18, 19]

* Received November 30, 2005.
1) This work was supported by the National Natural Science Foundation of China (grant No. 10231060), the Specialized Research Fund of Doctoral Program of Higher Education of China at No.20040319003 and the Graduates' Creative Project of Jiangsu Province, China.

proposes an approximate trust region method and an inexact line search approach, respectively, with the preconditioned modified gradient path and the optimal path.

Besides, it is well known that the impact of the selection of trust-region radius on computational behavior is quite notable. In [4] [10] [20], a new trust region subproblem, whose trust region radius is determined automatically by the information of the gradient and the Hessian or its approximation, is employed. With this new subproblem, an adaptive trust region method is constructed. Numerical results show that the adaptive trust region algorithm is quite competitive with the traditional trust region methods.

Motivated by the above ideas, in this paper we combine a new adaptive trust region method with curvilinear searches to form a new algorithm which is different from the algorithm in [18]. Particularly, the curvilinear path used here is a preconditioned modified gradient path.

This paper is organized as follows. Section 2 describes the adaptive nonmonotonic trust region algorithm with curvilinear searches. In section 3 we investigate the global convergence and the superlinear convergence rate. The numerical results of a set of standard test problems are presented in section 4. Finally, some concluding remarks are addressed in section 5.

2. Adaptive Nonmonotonic Trust Region Algorithm with Curvilinear Searches

As known, in the algorithms to solve the trust region subproblem approximately along curvilinear paths, the step is the solution of a subproblem in the form

$$\begin{aligned} \min \quad & q_k(\delta) = f_k + g_k^T\delta + \frac{1}{2}\delta^T B_k \delta, \\ \text{s.t.} \quad & \delta \in \Gamma_k, \quad \|\delta\| \le \Delta_k, \end{aligned} \tag{2.1}$$

where $f_k = f(x_k), g_k = \nabla f(x_k), \delta = x - x_k, B_k$ is either $\nabla^2 f(x_k)$ or its approximation, Δ_k the trust region radius, Γ_k a curvilinear path in either the full-dimensional space or a lower dimensional subspace, and $\|\cdot\|$ is the l_2 norm.

At the beginning of this section, we first recall the forming of the preconditioned modified gradient path and its relative properties.

The Bunch-Parlett factorization method [1] factorizes the matrix B_k into the form

$$P_k B_k P_k^T = L_k D_k L_k^T, \tag{2.2}$$

where P_k is a permutation matrix, L_k a unit lower triangular matrix and D_k a block diagonal matrix with 1×1 and 2×2 diagonal blocks. The matrices D_k and B_k have the same number of positive, zero and negative eigenvalues. Besides, the elements of L_k and L_k^{-1} are bounded, i.e., there exist positive constants c_1, c_2, c_3, c_4, which are independent of the matrix B_k, such that

$$c_1 \le \|L_k\| \le c_2, \quad c_3 \le \|L_k^{-1}\| \le c_4, \forall k. \tag{2.3}$$

Now we can use L_k and P_k to scale the variables, that is, we use the new variable

$$\widehat{\delta} = L_k^T P_k \delta,$$

and take the following trust region subproblem

$$\begin{aligned} \min \quad & \widehat{q}_k(\widehat{\delta}) = f_k + \widehat{g}_k^T\widehat{\delta} + \frac{1}{2}\widehat{\delta}^T D_k \widehat{\delta}, \\ \text{s.t.} \quad & \widehat{\delta} \in \Gamma_k, \quad \|\widehat{\delta}\| \le \Delta_k, \end{aligned} \tag{2.4}$$

where $\widehat{g}_k = L_k^{-1} P_k g_k$. In the above subproblem, $\widehat{\delta}$ is required to be within the trust region rather than $P_k^T L_k^{-T}\widehat{\delta}$, which improves the efficiency of calculation of the solution step.