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INVERSE MEDIUM SCATTERING PROBLEMS IN NEAR-FIELD OPTICS*[1)]

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Abstract

A regularized recursive linearization method is developed for a two-dimensional inverse medium scattering problem that arises in near-field optics, which reconstructs the scatterer of an inhomogeneous medium deposited on a homogeneous substrate from data accessible through photon scanning tunneling microscopy experiments. In addition to the ill-posedness of the inverse scattering problems, two difficulties arise from the layered background medium and limited aperture data. Based on multiple frequency scattering data, the method starts from the Born approximation corresponding to the weak scattering at a low frequency, each update is obtained via recursive linearization with respect to the wavenumber by solving one forward problem and one adjoint problem of the Helmholtz equation. Numerical experiments are included to illustrate the feasibility of the proposed method.

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Key words: Inverse medium scattering, Helmholtz equation, Near-field optics, Recursive linearization.

1. Introduction

Scattering problems are basic in many scientific research areas such as radar, sonar, geophysical exploration, medical imaging, and nondestructive testing [1–3]. In near-field optics, the scattering problems further involve wave fields containing evanescent components to improve resolution [4, 5], which arise naturally in diverse applications such as the imaging of biological samples, the inspection and manipulation of nano-electronic components in semiconductor technology, and the inspection and activation of nano-optical devices [6]. Near-field optics has attracted considerable attention as an effective approach to obtain images with subwavelength resolution [7].

Three important modalities that fall in the scenario of near-field optics are near-field scanning optical microscopy (NSOM) [8], total internal reflection microscopy (TIRM) [9, 10], and photon scanning tunneling microscopy (PSTM) [11,12]. In NSOM, the light source is transmitted through both the fiber and the small aperture at the tip of a probe. The probe is scanned over the sample in the near-field zone. The field scattered by the sample is then collected and measured in the far-field zone as a function of the probe position. In TIRM, the sample is

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illuminated by high spatial frequency evanescent plane waves, which may be generated by total internal reflection from a prism [13]. The scattered field is measured in the far zone of the sample as the direction of the incident wave is varied. In PSTM, the sample is illuminated by an evanescent field generated at the face of a prism (similar to TIRM), but the scattered field is detected via a tapered fiber probe in the near zone of the sample (as in NSOM). See [6] for an account of other modalities as basic experiments of near-field optics and associated scattering theories.

In all of the above mentioned modalities, it is desirable to solve the inverse scattering problem in order to reconstruct the sample from the measured data. Initial results in this direction have been reported in the cases of three-dimensional inhomogeneous media for all of the three modalities: NSOM [8] , TIRM [9], and more recently PSTM [11]. The basic idea is to develop an analytical solution technique for solving the linearized inverse scattering problem for each modality within the framework of the weak scattering approximation. Numerical solution of the nonlinear inverse problem is at present completely open. The main purpose of the present work is to explore the possibility of developing a regularized recursive linearization approach for solving the nonlinear inverse scattering problem in the modality of PSTM. It should be pointed out that we currently adopt the non-global approach, *i.e.*, the scattered field resulting from the interaction of the incident field with the sample is analyzed in the absence of the tip. This procedure overlooks the possible influence of the tip on the detected field. The global approach which takes into account the entire system is the subject of future work.

The remainder of this paper is organized as follows. The mathematical model of the scattering problems is introduced in Section 2. Based on the Lippmann–Schwinger integral equation in a two-layered background medium, an initial guess of the reconstruction is derived from the Born approximation corresponding to the weak scattering at a low frequency in Section 3. In Section 4, a regularized recursive linearization algorithm is proposed. Numerical examples are presented in Section 5. The paper is concluded with some general remarks and directions for future research in Section 6.

2. Model Problem

Consider an inhomogeneous sample deposited on a homogeneous substrate, usually a prism. The substrate is assumed to be relatively thick so that only one face needs to be considered, thus defining an interface between two half-spaces. The index of refraction in the lower half-space (substrate) has a constant value n_0. However, the index of refraction in the upper half-space varies within the domain of the sample but otherwise has a value of unity. The sample is illuminated from below (transmission geometry) by a time-harmonic plane wave, as shown in Fig. 2.1. Throughout, by assuming nonmagnetic materials and transverse electric polarization, the model PDE reduces to the two-dimensional Helmholtz equation.

More specifically, let an incoming plane wave $u^{\mathrm{i}} = \exp(i\alpha x_1 + i\beta x_2)$ be incident on the straight line $\{x_2 = 0\}$ from $\mathbb{R}^2_- = \{\mathbf{x} : x_2 < 0\}$, where

$$\alpha = n_0 k \sin\theta, \quad \beta = n_0 k \cos\theta, \quad \theta \in (-\pi/2, \pi/2),$$

and k is the free space wavenumber. The total field u satisfies the Helmholtz equation:

$$\Delta u + n^2 k^2 (1 + q) u = 0, \tag{2.1}$$