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# ON SOURCE ANALYSIS BY WAVE SPLITTING WITH APPLICATIONS IN INVERSE SCATTERING OF MULTIPLE OBSTACLES*

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**Abstract**

We study wave splitting procedures for acoustic or electromagnetic scattering problems. The idea of these procedures is to split some scattered field into a sum of fields coming from different spatial regions such that this information can be used either for inversion algorithms or for active noise control. Splitting algorithms can be based on general boundary layer potential representation or Green's representation formula. We will prove the unique decomposition of scattered wave outside the specified reference domain $G$ and the unique decomposition of far-field pattern with respect to different reference domain $G$. Further, we employ the splitting technique for field reconstruction for a scatterer with two or more separate components, by combining it with the point source method for wave recovery. Using the decomposition of scattered wave as well as its far-field pattern, the wave splitting procedure proposed in this paper gives an efficient way to the computation of scattered wave near the obstacle, from which the multiple obstacles which cause the far-field pattern can be reconstructed separately. This considerably extends the range of the decomposition methods in the area of inverse scattering. Finally, we will provide numerical examples to demonstrate the feasibility of the splitting method.

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## 1. Introduction

Inverse problems for acoustic and electromagnetic waves play an important role in many scientific and engineering applications. Medical imaging for example uses several techniques from the area of inverse problems as basic ingredients for medical examinations. Nondestructive testing employs inverse problems techniques for quality control. For a given incident wave, the impenetrable obstacle $D$ will generate a scattered wave outside $D$, which is in general governed by the Helmholtz equation for acoustic waves or Maxwell equations for electromagnetic waves. The scattered wave and its far-field pattern contain information about the scatterer $D$ such as

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the boundary shape and boundary type. The reconstruction of an obstacle $D$ from the far-field pattern of its scattered wave is one of the central research topics in inverse scattering theory, see for example [10] and the topical review [13].

There are three categories of shape reconstruction methods from far-field data of scattered waves. Firstly, there are iterative schemes, see, e.g., [5]. The second kind of methods use decomposition and optimization techniques, which firstly determine the scattered wave from its far-field pattern on a set outside of the obstacle and then update this surface such that the total field matches the boundary data by iteration procedure. This is a classical method with a long history [5, 7, 10]. The third kind of methods, which have been developed recently, constructs some indicator function of the boundary from the near field or its far-field pattern, respectively. Then the boundary shape is constructed from the point set where the indicator blows up in some way [2–4, 14, 15]. In most of these methods, the reconstruction of the scattered wave from its far-field pattern is of great importance.

By expressing the scattered wave outside $D$ in the form of a potential integral defined on $\partial D$, the direct scattering problem determines the scattered wave and its far-field using a density function which satisfies an integral equation on $\partial D$. This procedure is also applicable if $D$ contains multiple connected components ( [16]). Different methods for reconstructing the scattered field outside of $D$ have been developed, compare the literature given in [5]. Here we will employ and further develop the *potential method* of Kirsch-Kress [5] and the *point source method* of Potthast, Erhard, Liu, Chandler-Wilde, Lines and others (see [1, 10]). The methods reconstruct the scattered or total fields, respectively, outside of some auxiliary domain $G$. Both methods in their standard formulation have problems with reconstructing multiple obstacles $D = \bigcup D_j$ when the location of the obstacles is not known a-priori.

Following the potential approach, in principle we can still choose an approximate domain $G$ satisfying $G \supset \overline{D}$ such that we can compute the scattered wave outside $G$. The choice of $G$ can be specified from the knowledge of far-field pattern via *range test method* [12], where the solvability of some integral equation is tested (compare Section 2.4). However, it is a complex algorithmical task to use multiple auxiliary domains $G$ to reconstruct the field close to the boundary shapes $\partial D$ of the unknown scatterer $D$. Also, this leads to severe stability problems, since the ill-posedness of the equations under consideration depends on the curvature and non-convexity of the curves [8, 11].

Motivated by these problems, we present an efficient way to reconstruct the scattered wave from the far-field pattern caused by multiple obstacles. The basic idea is to *split the far-field pattern* into several parts which are essentially related to each obstacle. Correspondingly, the scattered wave is also decomposed. It is observed that our splitting *avoids any approximation* as for example employed for the Born approximation or physical optics approximation. Using this idea based on general potential theory or Green representation formula and combining it with the *point source method*, we propose a scheme which provides a reconstruction of the scattered wave at all points outside of some scatterer $D$ with several components. This *splitting method* enables the recovery of the scattered wave outside of multiple obstacles. The method proposed in this paper, except for its intrinsic importance in *wave recovery*, is also applicable to *shape reconstruction* for multiple obstacles.

This work is organized as follows. In Section 2, we firstly give the exact definition of scattered wave splitting and prove the *uniqueness* for this splitting for a given domain $G$. Then we establish two methods for the wave splitting, which are based on a *single-layer* approach and *Green's formula*, respectively. The uniqueness for both approaches is proven. To achieve