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STABLE SOLUTION OF TIME DOMAIN INTEGRAL EQUATION METHODS USING QUADRATIC B-SPLINE TEMPORAL BASIS FUNCTIONS*

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Abstract

This paper is concerned with stable solutions of time domain integral equation (TDIE) methods for transient scattering problems with 3D conducting objects. We use the quadratic B-spline function as temporal basis functions, which permits both the induced currents and induced charges to be properly approximated in terms of completeness. Because the B-spline function has the least support width among all polynomial basis functions of the same order, the resulting system matrices seem to be the sparsest. The TDIE formulations using induced electric polarizations as unknown function are adopted and justified. Numerical results demonstrate that the proposed approach is accurate and efficient, and no late-time instability is observed.

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Key words: TDIE methods, B-spline temporal basis functions, Transient scattering problems.

1. Introduction

Time domain integral equation (TDIE) methods have received much attention in several aspects in recent years, including stability, efficiency and application realms. Earlier studies of TDIE methods are concerned with the late-time instability [1-4] in transient scattering and radiation problems. The instability seriously impeded the popularity of the TDIE approaches. Fortunately, it has been found recently that this difficulty seems to be tractable by the proper choice of temporal basis functions [5-9]. The basis functions in [5-7] are compactly supported, while those in [8,9] are not which requires more memory and CPU time. In regarding to the efficiency of TDIE methods, the classical marching-on-in-time (MOT) scheme becomes more powerful by incorporating the fast Fourier transform (FFT) technique or plane wave time domain (PWTD) algorithm [10-13]. With the improvement of stability and efficiency, TDIE method is becoming a viable tool for simulation of complex microwave circuits containing nonlinear modules [12,13]. It is predicted that TDIE methods will have more applications for the wide-band analysis in many areas.

Though the TDIE method is likely to be the preferred solver for transient wave phenomenon, the stability problem is still a possible undermining factor, which is crucial for time domain simulation techniques. Although the use of the Lagrange interpolating temporal basis functions

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[5,6] can produce stable solutions under the conventional MOT framework, the rationality of using these functions is still unjustified. In fact, the first-order derivatives of these basis functions are not continuous; consequently, the second-order derivatives do not exist. This question is raised because both the current and its derivatives are required to calculate the scattered fields. In contrast, there is a reasonable justification for employing the quadratic B-spline function as the temporal basis functions [7], which ensures that any unknown function and its derivatives up to the second order can be approximated by the basis functions and their derivatives, respectively. In addition, because the B-spline functions are most compactly supported among all the polynomial basis functions of the same order, the resulting system matrices seem to be the sparest. Unconditional stability of the TDIE methods with quadratic B-spline temporal basis functions for wire problems has been reported in [7], and the purpose of this work is to demonstrate its suitability to arbitrary 3D conducting objects.

In the following parts, various TDIE formulations are described in Section 2, and numerical demonstrations are provided in Section 3. Some concluding remarks are given in Section 4.

2. Formulation

Consider a transient wave which is incident upon a PEC (perfectly-electrical-conducting) object as shown in Fig. 2.1, which induces a distribution of electric polarization $\mathbf{P}_s$ on the surface S. The induced currents and charges on the surface are related to $\mathbf{P}_s(\mathbf{r}', t')$ through

$$\mathbf{J}_s(\mathbf{r}', t') = \partial \mathbf{P}_s(\mathbf{r}', t')/\partial t' \quad \text{and} \quad \sigma_s(\mathbf{r}', t') = -\nabla'_s \cdot \mathbf{P}_s(\mathbf{r}', t'),$$

so that the continuity equation is satisfied automatically, i.e.,

$$\nabla'_s \cdot \mathbf{J}_s(\mathbf{r}', t') + \partial \sigma_s(\mathbf{r}', t')/\partial t' = 0.$$

The vector potential and scalar potential generated by the induced sources are

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int_S \frac{\mathbf{J}_s(\mathbf{r}', t - R/c)}{R} dS', \tag{2.1}$$

$$\phi(\mathbf{r}, t) = \frac{1}{4\pi\varepsilon_0} \int_S \frac{\sigma_s(\mathbf{r}', t - R/c)}{R} dS'. \tag{2.2}$$

The scattered fields can be expressed by the potentials as

$$\mathbf{E}^s = -\frac{\partial \mathbf{A}}{\partial t} - \nabla\phi = -\frac{1}{4\pi\varepsilon_0} \int_S \left[\frac{\partial^2 \mathbf{P}_s(\mathbf{r}', t - R/c)}{R\partial(ct)^2} - \nabla \frac{[\nabla'_s \cdot \mathbf{P}_s(\mathbf{r}', t')]_{t'=t-R/c}}{R} \right] dS', \tag{2.3}$$

$$\mathbf{H}^s = \frac{1}{\mu_0} \nabla \times \mathbf{A} = -\frac{1}{\eta_0} \frac{1}{4\pi\varepsilon_0} \int_S \frac{\hat{R}}{R} \times \left[\frac{\partial \mathbf{P}_s(\mathbf{r}', t - R/c)}{R\partial(ct)} + \frac{\partial^2 \mathbf{P}_s(\mathbf{r}', t - R/c)}{\partial(ct)^2} \right] dS', \tag{2.4}$$

where $c = 1/\sqrt{\varepsilon_0 \mu_0}$ is the velocity of light in vacuum and $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ is the intrinsic impedance of the free space. The boundary conditions for a PEC surface are $\hat{n} \times (\mathbf{E}^i + \mathbf{E}^s) = \mathbf{0}$ and $\hat{n} \times (\mathbf{H}^i + \mathbf{H}^s) = \mathbf{J}_s = \partial \mathbf{P}_s/\partial t$, which by virtue of (2.3)-(2.4) become

$$\hat{n} \times \frac{1}{4\pi\varepsilon_0} \int_S \left[\frac{\partial^2 \mathbf{P}_s(\mathbf{r}', t - R/c)}{R\partial(ct)^2} - \nabla \frac{[\nabla'_s \cdot \mathbf{P}_s(\mathbf{r}', t')]_{t'=t-R/c}}{R} \right] dS' = n \times \mathbf{E}^i, \tag{2.5}$$

$$\frac{1}{2} \frac{\partial \mathbf{P}_s(\mathbf{r}, t)}{\partial t} + \hat{n} \times \frac{c}{4\pi} P.V. \int_S \frac{\hat{R}}{R} \times \left[\frac{\partial \mathbf{P}_s(\mathbf{r}', t - R/c)}{R\partial(ct)} + \frac{\partial^2 \mathbf{P}_s(\mathbf{r}', t - R/c)}{\partial(ct)^2} \right] dS' = \hat{n} \times \mathbf{H}^i, \tag{2.6}$$