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A FINITE ELEMENT METHOD WITH PERFECTLY MATCHED ABSORBING LAYERS FOR THE WAVE SCATTERING BY A PERIODIC CHIRAL STRUCTURE [*1)]

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Abstract

Consider the diffraction of a time-harmonic wave incident upon a periodic chiral structure. The diffraction problem may be simplified to a two-dimensional one. In this paper, the diffraction problem is solved by a finite element method with perfectly matched absorbing layers (PMLs). We use the PML technique to truncate the unbounded domain to a bounded one which attenuates the outgoing waves in the PML region. Our computational experiments indicate that the proposed method is efficient, which is capable of dealing with complicated chiral grating structures.

Mathematics subject classification: 35Q60, 65L60, 78A45.
Key words: Chiral media, Perfectly matched layer, Grating optics.

1. Introduction

Consider a time-harmonic electromagnetic plane wave incident on a periodic chiral structure which is periodic in x_1- direction and invariant in x_3- direction. The medium inside the structure is chiral and separates two homogeneous regions. The scattering problem may be simplified to a two-dimensional one. In this paper, we propose and analyze a finite element method with perfectly matched absorbing layers for the scattering problem.

Recently, there has been a considerable interest in the study of scattering and diffraction by chiral media. In general, the electromagnetic fields inside the chiral medium are governed by Maxwell equations together with the Drude-Born-Fedorov equations in which the electric and magnetic fields are coupled. The chiral media is characterized by the electric permittivity ε, the magnetic permeability μ and the chirality measure β. On the other hand, periodic structures (gratings) have received increasing attentions through the years because of importance applications in integrated optics, optical lenses, et al.

Scattering theory in chiral structures has recently received considerable attention in the applied mathematical community. We refer to Ammari and Bao [1], Ammari and Nédélec [2] for the existence and uniqueness to the scattering problem for bi-periodic chiral media. A good introduction to the electromagnetic diffraction through chiral structures can be found in Lakhtakia [3] and Lakhtakia, Varadan and Varadan [4] (non-periodic chiral structures).

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This work is a continuation of our recent analysis of diffraction problem of Zhang and Ma [5]. In [5], we simplify the diffraction problem into a two-dimensional one and established the well-posedness. We also propose a finite element method and give the numerical analysis for the scattering problem.

The purpose of this paper is to develop efficient numerical methods for solving the scattering problem. In doing so, the main difficulty is to truncate the domain into a bounded computational domain. The finite element method studied in [5] is based on variational formulation on a bounded domain, with periodic condition in the x_1-direction and the transparent boundary condition on the top and bottom boundaries. The transparent boundary condition is obtained by insisting that the solutions be composed of bounded outgoing plane waves, plus the incident wave in the domain above the structure. The derived transparent boundary condition is represented as a quasi-differential operator and is nonlocal. In practical computations, the infinite series must be truncated. In [6], for the wave scattering by periodic (achiral) structures, Chen and Wu use perfectly matched layer (PML) technique to deal with the difficulty. In this paper, we will develop the PML method to solve the scattering problem for chiral structures.

Under the assumption that the exterior solution is composed of outgoing waves only, the basic idea of the PML technique is to surround the computational domain with a finite thickness layer of a specially designed model medium, which would either slow down or attenuate all the waves that propagate from inside the computational domain. Since the work of Berenger [7], which proposed a PML for use with the time dependent Maxwell equations, various constructions of PML absorbing layers have been proposed and studied in the literature. We refer to Turkel and Yefet [8] for a review on various proposed models, and Lassas and Somersalo [9] for the study of mathematical properties of the PML equations.

The layout of the paper is as follows. In the next section, we state the model problem and a variational formulation. We discuss the energy distribution of diffracted waves in Section 3. In Section 4, we introduce our PML formulation, and establish the existence, uniqueness and convergence of the PML formulation. Finally, in Section 5, we present several numerical examples to illustrate the advantages of our method.

2. The Scattering Problem

Let us consider the propagation of time-harmonic electromagnetic waves. The electromagnetic fields are governed by the time-harmonic (time dependence $e^{-i\omega t}$) Maxwell's equations

$$\nabla \times \mathbf{E} - i\omega \mathbf{B} = 0, \tag{2.1}$$

$$\nabla \times \mathbf{H} + i\omega \mathbf{D} = 0, \tag{2.2}$$

where $\mathbf{E}, \mathbf{H}, \mathbf{D}$ and $\mathbf{B}$ denote the electric field, the magnetic field, the electric and magnetic displacement vectors in $\mathbb{R}^3$, respectively. For chiral media, $\mathbf{E}, \mathbf{H}, \mathbf{D}$ and $\mathbf{B}$ satisfy with the Drude-Born-Fedorov constitutive equations:

$$\mathbf{D} = \varepsilon(x)(\mathbf{E} + \beta(x)\nabla \times \mathbf{E}), \tag{2.3}$$

$$\mathbf{B} = \mu(x)(\mathbf{H} + \beta(x)\nabla \times \mathbf{H}), \tag{2.4}$$