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MULTISTEP FINITE VOLUME APPROXIMATIONS TO THE TRANSIENT BEHAVIOR OF A SEMICONDUCTOR DEVICE ON GENERAL 2D OR 3D MESHES *1)

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Abstract

In this paper, we consider a hydrodynamic model of the semiconductor device. The approximate solutions are obtained by a mixed finite volume method for the potential equation and multistep upwind finite volume methods for the concentration equations. Error estimates in some discrete norms are derived under some regularity assumptions on the exact solutions.

Mathematics subject classification: 65N30, 65M25.
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1. Introduction

Let us consider a system of equations describing the mobil carrier transport in a semiconductor device in a bounded domain $\Omega \in R^d$, $d = 2, 3$:

$$-\Delta v = \nabla \cdot u = \alpha[p - e + N(x)], \tag{1.1}$$

$$\frac{\partial e}{\partial t} = \nabla \cdot [D_e(x)\nabla e - \mu_e(x)e\nabla v] - R_1(e,p), \tag{1.2}$$

$$\frac{\partial p}{\partial t} = \nabla \cdot [D_p(x)\nabla p + \mu_p(x)p\nabla v] - R_2(e,p). \tag{1.3}$$

The above system is a hydrodynamic model of the semiconductor device. Three unknowns are the electrostatic potential v, the electron mobile charge density e, and the hole mobile charge density p. $u = -\nabla v$ is the electric intensity. α is a constant related to the magnitude of electronic charge and the dielectric permittivity. $N(x) = N_D(x) - N_A(x)$, where $N_D(x)$ and $N_A(x)$ denote the donor and acceptor impurity respectively. The diffusion coefficients $D_s(x)$ $(s = e, p)$ are related to the mobilities $\mu_s(x)$ by the relation $D_s(x) = U_T\mu_s(x)$, where U_T is the thermal voltage. The recombination terms $R_i(e,p), i = 1, 2$ are Lipschitz continuous with the Lipschitz constant λ. All the coefficients appeared in (1.1)–(1.3) are positive and bounded, and $\mu_s \geq \mu_* > 0, D_s \geq D_* > 0, s = e, p$, where μ_* and D_* is positive constants.

The equations can be completed by the following initial and boundary conditions

$$e(x,0) = e_0(x),\ p(x,0) = p_0(x), \tag{1.4}$$

$$v = 0,\ e = 0,\ p = 0,\ x \in \partial\Omega,\ t \in (0,T]. \tag{1.5}$$

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There have existed many works on the numerical solution of the above system. In [5], a finite difference method was constructed for one or two dimensional cases and the convergence analysis was given. Numerical procedures based on a mixed finite element method for the potential equation and finite element methods for the mobile charge density equations were first presented in [4, 7]. The method was then applied to a mixed initial boundary model in [12], where under some less smoothness assumptions on the exact solution, a priori error estimates were obtained. In [13], two kinds of finite element schemes, one being partly linear and another being nonlinear, were formulated and the existence of the approximate solutions was proved for both cases. The convergence analysis for the nonlinear scheme was presented in [14]. Some exponentially converging box methods, named Scharfetter-Gummel methods, were used in [10] to treat two and three dimensional semiconductor device problems. The stability of the methods and error estimates for the Slotboom variables are derived. Recently, characteristic finite element methods have been presented in [11] to avoid nonphysical oscillation and optimal error estimates were obtained there.

Finite volume method is a discretization tool used extensively in the computations for conservation laws. The method is suitable in handling general domains, which can keep local conservation properties of the numerical fluxes. We refer to [3, 6, 8, 9] and the references therein for some details. In this paper, we study a finite volume method for the semiconductor devices in multi–dimensions. We use a mixed finite volume method to treat the elliptic equation (1.1) and upwind finite volume methods to treat the convection–diffusion Eqs. (1.2)-(1.3). A multistep time discretization is considered to enhance the accuracy in temporal direction. Under the assumption that the exact solutions possess enough regularity we derive the optimal error estimates in discrete norms for the scheme.

The rest of the paper is organized as follows. In Section 2, we introduce the admissible meshes and some necessary notation. Section 3 is devoted to formulating a fully discrete finite volume scheme for Eqs. (1.1)-(1.5). In Section 4, we derive the priori error estimates for the finite volume scheme under some regularity assumptions on the exact solutions.

Throughout this paper, we use C and ϵ to denote a general positive constant and a general positive small constant, respectively, not necessarily the same in different places.

2. Meshes and Notations

Definition 2.1. (Admissible meshes) *An admissible mesh T_h of Ω is given by a family of control volumes, which are open polygonal (or polyhedral) subsets of Ω. A family $\mathcal{E}$ of subsets of $\overline{\Omega}$ contained in hyper-planes of R^d with strictly positive measure (the edges of the mesh), and a family of discrete points in Ω satisfying the following properties:*

1. *The closure of the union of all control volumes is $\overline{\Omega}$.*
2. *For any $K \in T_h$, there exists a subset $\mathcal{E}_K \subseteq \mathcal{E}$, such that $\partial K = \overline{K} \backslash K = \cup_{\sigma \in \mathcal{E}_K} \overline{\sigma}$. Furthermore, $\mathcal{E} = \cup_{K \in T_h} \mathcal{E}_K$.*
3. *For any $(K, L) \in T_h^2$ with $K \neq L$, either $\overline{K} \cap \overline{L} = 0$ or $\overline{K} \cap \overline{L} = \overline{\sigma}$. Then, we denote by $\sigma = K|L$.*
4. *The family of discrete points $\{x_K\}_{K \in T_h}$ is such that $x_K \in K$ and, if $\sigma = K|L$, it is assumed that the straight line $\overline{x_K x_L}$ is orthogonal to σ.*

Let h denote the space step of the mesh T_h. For any $K \in T_h$ and $\sigma \in \mathcal{E}$, we denote by $m(K)$ the measure of K and $m(\sigma)$ the measure of the edge σ. If $\sigma \in \mathcal{E}_K$, we denote $d_{K,\sigma}$ the Euclidean