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A FLEXIBLE PRECONDITIONED ARNOLDI METHOD FOR SHIFTED LINEAR SYSTEMS*

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Abstract

We are interested in the numerical solution of the large nonsymmetric shifted linear system, $(A+\alpha I)x=b$, for many different values of the shift α in a wide range. We apply the Saad's flexible preconditioning technique [14] to the solution of the shifted systems. Such flexible preconditioning with a few parameters could probably cover all the shifted systems with the shift in a wide range. Numerical experiments report the effectiveness of our approach on some problems.

Mathematics subject classification: 65F10, 65Y20.
Key words: Shifted linear systems, Arnoldi method, Flexible preconditioning.

1. Introduction

We are interested in the numerical solution of the following large nonsymmetric shifted linear system,

$$(A+\alpha_j I)x(\alpha_j)b, \qquad j=1,\cdots,s, \tag{1.1}$$

for many, possibly a few hundreds, different values of the shift α_j in a wide range, all available simultaneously. This problem arises in many engineering applications like in quantum chromodynamics [8], electromagnetics [12], structural dynamics [5,17], wave propagation [15] and control theory [4]. The traditional approach to this problem is to factorize $A+\alpha_j I$ and solve (1.1) by backtransformation for each α_j. This can be quite expensive when s is large. Now the Krylov subspace methods is a popular approach to solve (1.1); see e.g. [4,7,9,10,17],since these are invariant with respect to shift α_j. More precisely, the Krylov subspace satisfies $\mathcal{K}_m(A,b)=\mathcal{K}_m(A+\alpha_j I,b)$, for any α_j. Hence, all approximation solutions can be sought in a single subspace generated by the constant coefficient matrix A.

However, convergence may be slow if the coefficient shifted matrix $A+\alpha_j I$ has unfavorable spectral properties. Applying an efficient preconditioner for the shifted systems (1.1) is necessary and important. Some attempts have been made in the past, e.g. polynomial preconditioning, which preserves the shifted form [6], and approximate inverse preconditioner for each shifted matrix $A+\alpha_j I$ by cheaply modifying an existing sparse approximate inverse preconditioner for A [3].

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In [13], the preconditioning matrix $(A+\sigma I)^{-1}$ with a fixed reference σ is used. This leads to solve the preconditioned shifted systems

$$(A+\sigma I)^{-1}(A+\alpha_j I)x(\alpha_j)=(A+\sigma I)^{-1}b, \qquad j=1,\cdots,s. \tag{1.2}$$

This preconditioning approach maintains the shift-invariance properties of the Krylov subspace, since

$$(A+\sigma I)^{-1}(A+\alpha_j I)=I+(\alpha_j-\sigma)(A+\sigma I)^{-1}. \tag{1.3}$$

Thus, all the approximation solutions can be sought in one Krylov subspace generated by the matrix $(A+\sigma I)^{-1}$. Meerbergen [13] analyzed the spectrum of the preconditioned matrix $(A+\sigma I)^{-1}(A+\alpha_j I)$. The nice features are that the preconditioner is well suited for values of α_j near the reference value σ. However, it is difficult for only one reference value σ to cover many different values α_j in a wide range.

Based on Saad's flexible preconditioning idea, the FOM/GMRES method with a variable preconditioning for solving the shifted systems (1.1) is presented in this paper. The method allows us to incorporate the different preconditioner, e.g. $(A+\sigma_i I)^{-1}$ with different σ_i in our problem, into the Arnoldi procedure when constructing a projective subspace. It is possible for such single projective subspace with a few different preconditioning matrices $(A+\sigma_i I)^{-1}$ to cover all the different values α_j in a wide range. Although such projective subspace is not a Krylov subspace, it is still invariant with respect to the shift α_j, or say that the subspace is independent on the shift α_j. Hence, all the approximation solutions can still be sought in the single projective subspace generated by the preconditioned matrices $(A+\sigma_i I)^{-1}$. Numerical experiments report the effectiveness of our approach on some problems.

The remainder of the paper is organized as follows. In Section 2, we first change the left version of Meerbergen's preconditioner to the right version. We then present the Arnoldi method with a flexible preconditioning. In Section 3, we discuss some implementation issues of the algorithm. Numerical experiments are shown in Section 4.

2. Projective Subspace with Preconditioning

In this section, we first present the right version of Meerbergen's preconditioning to construct a flexible preconditioning, and then based on it, we establish FOM/GMRES method with a flexible preconditioning for solving the shifted linear systems (1.1).

2.1. Right Preconditioning

We employ the right preconditioner $(A+\sigma I)^{-1}$ to the shifted system (1.1),

$$(A+\alpha_j I)(A+\sigma I)^{-1}\tilde{x}(\alpha_j)=b, \qquad j=1,\cdots,s, \tag{2.1}$$

where $\tilde{x}(\alpha_j)=(A+\sigma I)x(\alpha_j)$. Since

$$(A+\alpha_j I)(A+\sigma I)^{-1}=I+(\alpha_j-\sigma)(A+\sigma I)^{-1}, \tag{2.2}$$

Krylov subspace $\mathcal{K}_m((A+\sigma I)^{-1},b)$ generated by $(A+\sigma I)^{-1}$ is invariant with respect to shift α_j, *i.e.*, $\mathcal{K}_m((A+\sigma I)^{-1},b)=\mathcal{K}_m((A+\alpha_j I)(A+\sigma I)^{-1},b)$, for any α_j. Hence, all preconditioned shifted systems(2.1) can be projected onto a single approximation subspace $\mathcal{K}_m((A+\sigma I)^{-1},b)$. The following Arnoldi procedure builds an orthogonal basis of the Krylov subspace $\mathcal{K}_m((A+\sigma I)^{-1},b)$.