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RESERVOIR DESCRIPTION BY USING A PIECEWISE CONSTANT LEVEL SET METHOD*

Hongwei Li
Center for Integrated Petroleum Research, University of Bergen, Norway;
Department of Mathematics, Capital Normal University, Beijing 100037, China
Email: Hongwei.li@cipr.uib.no
Xuecheng Tai
Department of Mathematics, University of Bergen, Norway;
Division of Mathematical Sciences, School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore
Email: Tai@mi.uib.no
Sigurd Ivar Aanonsen
Center for Integrated Petroleum Research, University of Bergen, Norway;
Department of Mathematics, University of Bergen, Norway
Email: Sigurd.Aanonsen@cipr.uib.no

Dedicated to Professor Junzhi Cui on the occasion of his 70th birthday

Abstract

We consider the permeability estimation problem in two-phase porous media flow. We try to identify the permeability field by utilizing both the production data from wells as well as inverted seismic data. The permeability field is assumed to be piecewise constant, or can be approximated well by a piecewise constant function. A variant of the level set method, called Piecewise Constant Level Set Method is used to represent the interfaces between the regions with different permeability levels. The inverse problem is solved by minimizing a functional, and TV norm regularization is used to deal with the ill-posedness. We also use the operator-splitting technique to decompose the constraint term from the fidelity term. This gives us more flexibility to deal with the constraint and helps to stabilize the algorithm.

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1. Introduction

History matching, i.e, the process of tuning uncertain properties to match dynamic data is an important part of reservoir flow modelling. The flow in the reservoir (porous medium) can be modelled by multiphase flow equations. If the physical (rock) properties of the reservoir, such as porosity, permeability etc. are known, then we can predict the pressure and saturation distributions by solving the flow equations. However, such information usually are only available at wells. On the other hand, the production data from wells, as well as seismic data might be available, which can provide sparsely distributed pressure and saturation information. Based on these data, we should be able to get some information about the rock properties. However,

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because of the sparsity of data, one shouldn't expect to identify the fine details. We focus on permeability estimation in this paper, and all the other rock properties, such as porosity, viscosity etc, are supposed to be known.

Permeability estimation is an inverse problem, which is ill-posed and usually needs regularization. In our case, the ill-posedness is worse because of the insufficient observation data. To overcome this difficulty, we give up pursuing the fine resolution of the permeability field, and adopt the common used zonation regularization strategy. To do so, the parameter field is assumed to be piecewise constant. This reduces the solution space, so that non-feasible solutions should be avoided. Accordingly, TV (Total Variation) norm regularization will be used to control the shape of the curves separating different constant permeability regions, which has the ability to preserve sharp interfaces [1].

When solving curve evolution problems, the level set method is an natural choice, because of its excellent ability to deal with topological changes, such as breakings or mergings, in a natural and efficient way. Since the permeability estimation problem considered here involves identifying the curves separating different regions, we will also use level set method to represent the interfaces implicitly, and solve the inverse problem by evolving the level set functions.

The original level set method was proposed by Osher and Sethian in [2]. Recently, the level set method has been extended and used for various inverse problems [3–7]. Recently, some variants of the Osher-Sethian level set method have been proposed [8, 9]. A variant of level set method, called binary level set method, was proposed in [9, 10] and used for image processing. It simplifies the original level set method, and may gain some advantages for certain problems. This method has been tested for our problem for the case that the permeability field consists of two regions of constant values, see Nielsen et. al. [11]. By binary level set method, the zonation structure is implicitly represented by binary level set functions. One binary level set function can represent two regions. With more than two regions, we need to use multiple binary level set functions, see [8, 12]. This is just the multiple (traditional) level set method framework, proposed by Chan et.al. [13, 14]. The piecewise constant level set method [8], is another variant of the level set method. However, piecewise constant level set method has the ability to deal with multiple regions using just one level set function. This is simpler than the multiple binary level set method.

In this paper, we try to identify the permeability field with zonation structure by using the piecewise constant level set method of [8]. The permeability field is assumed to be piecewise constant, or can be approximated well by piecewise constant functions. The observation data available is production data from wells and inverted, time-lapse seismic data.

The remainder of this paper is organized as follows. In Section 2, the inverse problem and its corresponding minimization problem are set up. In Section 3, the piecewise constant level set method is introduced and incorporated into the formulation of the minimization problem. The numerical algorithm is described in Section 4. Numerical experiments are presented in Section 5. Section 6 is for conclusion and remarks.

2. The Inverse and Minimization Problem

We consider incompressible, two dimensional two phase flow (oil and water) in a porous medium with isotropic permeability and zero capillary pressure.

$$\Phi(x)\frac{\partial S_o}{\partial t} - \nabla\cdot\left(\frac{\kappa(x)\kappa_{ro(S_o)}}{\mu_o}\nabla p\right) = f_o(x), \tag{2.1}$$