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AN AFFINE SCALING INTERIOR ALGORITHM VIA CONJUGATE GRADIENT PATH FOR SOLVING BOUND-CONSTRAINED NONLINEAR SYSTEMS*

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Abstract

In this paper we propose an affine scaling interior algorithm via conjugate gradient path for solving nonlinear equality systems subject to bounds on variables. By employing the affine scaling conjugate gradient path search strategy, we obtain an iterative direction by solving the linearize model. By using the line search technique, we will find an acceptable trial step length along this direction which is strictly feasible and makes the objective function nonmonotonically decreasing. The global convergence and fast local convergence rate of the proposed algorithm are established under some reasonable conditions. Furthermore, the numerical results of the proposed algorithm indicate to be effective.

Mathematics subject classification: 90C30, 65K05.
Key words: Conjugate gradient path, Interior points, Affine scaling.

1. Introduction

In this paper we use an affine scaling interior conjugate gradient path method to analyze the solution of nonlinear systems subject to the bound constraints on variable:

$$F(x) = 0, \quad x \in \Omega = \{\ x \mid l \le x \le u\ \}, \tag{1.1}$$

where $F : \mathcal{X} \to \Re^n$ is a given continuously differentiable mapping and $\mathcal{X} \subseteq \Re^n$ is an open set containing the n-dimensional box constraint Ω. The vector $l \in (\Re \cup \{-\infty\})^n$ and $u \in (\Re \cup \{+\infty\})^n$ are specified lower and upper bounds on the variables such that

$$\mathrm{int}(\Omega) \stackrel{\mathrm{def}}{=} \{\ x \mid l < x < u\ \}$$

is nonempty, where $l < u$. The problem (1.1) arises naturally in systems of equations modeling real-life problems when not all the solutions of the model have physical meaning. For example, cross-sectional properties of structural elements, dimensions of mechanical linkages, concentrations of chemical species, etc., are modeled by nonlinear equations where Ω is the positive orthant of $\Re^n$ or a closed box constraint. In the classical methods for solving the unconstrained nonlinear equations (1.1) when the function $F(x)$ is a continuously differentiable function, the Newton method or quasi-Newton method can be used. These methods by using the Jacobian or version of Newton's method often solve the unconstrained problem (1.1), which is known to

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have locally very rapid convergence (see, e.g., [3, 4]). However, the Newton methods used for smooth systems (1.1) does not ensure global convergence, that is, the convergence is only local. Other methods for solving (1.1) can be found in, e.g., [11, 14].

Many papers about affine-scaling algorithm for solving problems appeared during the last few years. Sun in [9] gave a convergence proof for an affine-scaling algorithm for convex quadratic programming without nondegeneracy assumptions, and Ye [12] introduced affine scaling algorithm for nonconvex quadratic programming. Classical trust-region Newton method for solving the nonlinear system (1.1) and the affine scaling double trust-region approach for solving the bounded constrained optimization problems are given in [2]. Recently, Bellavia et al. in [1] further extended the idea and presented an affine scaling trust-region approach for solving the bounded-constrained smooth nonlinear systems (1.1). However, the search direction generated in trust-region subproblem must satisfy strict interior feasibility which results in computational difficulties. In this paper, we introduce an affine scaling interior algorithm via conjugate gradient path to solve the bound-constrained nonlinear systems (1.1).

In order to describe and design the affine scaling interior conjugate gradient path algorithm for solving the bound-constrained smooth equations (1.1), we first introduce the squared Euclidean norm of linearize model of the unconstrained systems (1.1) and the augmented quadratic affine scaling model, and state the affine scaling conjugate gradient path with backtracking interior point technique for the bound-constrained nonlinear equations in Section 2. In Section 3, we prove the global convergence of the proposed algorithm. We discuss some further convergence properties such as strong global convergence and characterize the order of local convergence of the Newton method in terms of the rates of the relative residuals in Section 4. Finally, the results of numerical experiments of the proposed algorithm are reported in Section 5.

2. Algorithm

In this section we describe and design the affine scaling conjugate gradient strategy in association with nonmonotonic interior point backtracking technique for solving the bound-constrained nonlinear minimization transformed by the bound-constrained systems (1.1) and present an interior point backtracking technique which enforces the variable generating strictly feasible interior point approximations to solution of the bound-constrained nonlinear minimization.

Bellavia et al. in [1] presented the affine scaling trust-region approach scheme. The basic idea is based on the trust region subproblem at the kth iteration

$$\begin{aligned} \min \quad & q_k(d) \stackrel{\text{def}}{=} \frac{1}{2}\|F_k^{'}d + F_k\|^2 = \frac{1}{2}\|F_k\|^2 + F_k^T F_k^{'} d + \frac{1}{2} d^T (F_k^{'T} F_k^{'}) d \\ \text{s.t.} \quad & \|D_k d\| \le \Delta_k, \end{aligned} \tag{2.1}$$

where F' is the Jacobi matrix of F, Δ_k is the trust region radius and $q_k(d)$ is trusted to be an adequate representation of the merit function

$$f(x) \stackrel{\text{def}}{=} \frac{1}{2}\|F(x)\|^2. \tag{2.2}$$

The scaling matrix $D_k = D(x_k)$ arises naturally from examining the first-order necessary conditions for the bound-constrained nonlinear minimization transformed by the bound-constrained problem (1.1), where $D(x)$ is the diagonal scaling matrix such that

$$D(x) \stackrel{\text{def}}{=} \text{diag}\{|v^1(x)|^{-\frac{1}{2}}, \cdots, |v^n(x)|^{-\frac{1}{2}}\} \tag{2.3}$$