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MULTIGRID METHOD FOR A MODIFIED CURVATURE DRIVEN DIFFUSION MODEL FOR IMAGE INPAINTING*

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Abstract

Digital inpainting is a fundamental problem in image processing and many variational models for this problem have appeared recently in the literature. Among them are the very successfully Total Variation (TV) model [11] designed for local inpainting and its improved version for large scale inpainting: the Curvature-Driven Diffusion (CDD) model [10]. For the above two models, their associated Euler Lagrange equations are highly nonlinear partial differential equations. For the TV model there exists a relatively fast and easy to implement fixed point method, so adapting the multigrid method of [24] to here is immediate. For the CDD model however, so far only the well known but usually very slow explicit time marching method has been reported and we explain why the implementation of a fixed point method for the CDD model is not straightforward. Consequently the multigrid method as in [Savage and Chen, *Int. J. Comput. Math.,* 82 (2005), pp. 1001-1015] will not work here. This fact represents a strong limitation to the range of applications of this model since usually fast solutions are expected. In this paper, we introduce a modification designed to enable a fixed point method to work and to preserve the features of the original CDD model. As a result, a fast and efficient multigrid method is developed for the modified model. Numerical experiments are presented to show the very good performance of the fast algorithm.

Mathematics subject classification: 68U10, 65F10, 65K10.
Key words: Image inpainting, Variational models, Regularization, Multilevel methods.

1. Introduction

Image inpainting has been defined as the process of reconstituting the missing or damaged portions of an image, in order to make it more legible and to restore its unity. The aim of inpainting is then to modify an image in a way that is non-detectable for an observer who does not know the original image [2].

There are a variety of reasons why images can have damaged parts, for instance because of some physical degradation like aging, weather or intentional scratching. Not only that, we also would like to recover parts of objects of an image occluded by other objects or to reconstruct parts that have been missing due to digital communication processes. We can imagine a number of applications of this technique: among the most known are the restoration of old pictures with scratches or missing patches [2], text removal, digital zooming and superresolution [11], error concealment [14], disocclusion in computer vision, X-Ray CT artifacts reduction [18], and the long list continues. Inpainting techniques deal with these kinds of problems trying

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to reconstruct in the best possible way the missing or damaged parts of an image from the available information.

The mathematical interest in this field became increasingly active in the last decade since the very first works on image interpolation by Mumford, Nitzberg and Shiota [22], Masnou and Morel [21] and Caselles, Morel and Sbert [5]. However it was the pioneering work of Bertalmio *et al* [2] who proposed an algorithm to imitate the work of inpainting artists who manually restore old damaged pictures which mainly motivated all the subsequent research in this field [11, 12]. This algorithm cleverly transports a smoothness image measure (namely the Laplacian of the image) along the level lines (contours of the same image intensity) directed into the inpainting domain; in their paper, they also showed that some intermediate steps of anisotropic diffusion are necessary to avoid blurring of edges. This algorithm was created mostly intuitively but later on turned out to be closely related to the Navier-Stokes equation, as showed by Bertozzi *et al*; see [4]. Since then, many other authors have proposed different models for digital inpainting.

Chan and Shen [11] introduced the Total Variation (TV) model for local inpainting based on the celebrated total variation based image denoising model of Rudin, Osher and Fatemi [23]. Later on the same authors modified this model to improve its performance for large scale inpainting, and created the so-called Curvature-Driven Diffusion (CDD) model [10]. Furthermore they, together with Kang, introduced a higher order variational model [9] based on the Euler's elastica which connects the level lines by using Euler elastica curves [19] instead of using straight lines as the TV model does. Unfortunately for the latter two models there appear to exist no fast methods to find the numerical solution. The aim of this paper is to develop a fast multigrid algorithm for the CDD model.

A related inpainting model was proposed by Esedoglu and Shen [16] and is based on the very successfully Mumford-Shah image segmentation model. This model is also good for local inpainting but shares the same problem as the TV model in that it cannot reconnect separated parts of broken objects far apart. To fix this problem, the same authors of [16] proposed the Mumford-Shah-Euler inpainting model which in the same fashion of the Euler's elastica model uses the information encoded in the curvature to reconnect smoothly the level lines. More recently, in separate works, Bertozzi, Esedoglu and Gillete [3] proposed a model to inpaint binary images based on the Cahn-Hilliard equation and Grossauer and Scherzer [17] proposed a model based on the complex Ginzburg-Landau equation. It remains to develop fast multigrid methods for these models.

Each one of the above models has its own particular features which may not suit all applications. However as rightly remarked in [13] one of the most interesting open problems in digital inpainting (whatever the model) is the fast and efficient digital realization. The new multigrid method for the CDD model is our first step in this direction.

We remark that measuring the quality of restoration is non-trivial as physical perceptions can be different and the 'true' solutions may not be unique [12]. In our tests, we have chosen one perception as the true solution.

The rest of this paper is organized as follows. Section 2 introduces the image inpainting problem and two variational models, followed by the review of a commonly-used numerical method in Section 3. Section 4 describes first the modified CDD model and then the framework of a nonlinear multigrid method with emphasis on two smoothers, global and local. A local Fourier analysis is shown to give an indication of the effectiveness of both smoothers. Finally Section 5 presents some testing results illustrating the effectiveness of the modified model and the associated multigrid method.