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THREE-DIMENSIONAL NUMERICAL LOCALIZATION OF IMPERFECTIONS BASED ON A LIMIT MODEL IN ELECTRIC FIELD AND A LIMIT PERTURBATION MODEL*

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Abstract

From a limit model in electric field obtained by letting the frequency vanish in the time-harmonic Maxwell equations, we consider a limit perturbation model in the tangential boundary trace of the curl of the electric field for localizing numerically certain small electromagnetic inhomogeneities, in a three-dimensional bounded domain. We introduce here two localization procedures resulting from the combination of this limit perturbation model with each of the following inversion processes: the Current Projection method and an Inverse Fourier method. Each localization procedure uses, as data, a finite number of boundary measurements, and is employed in the single inhomogeneity case; only the one based on an Inverse Fourier method is required in the multiple inhomogeneities case. Our localization approach is numerically suitable for the context of inhomogeneities that are not purely electric. We compare the numerical results obtained from the two localization procedures in the single inhomogeneity configuration, and describe, in various settings of multiple inhomogeneities, the results provided by the procedure based on an Inverse Fourier method.

Mathematics subject classification: 65N21, 65N30, 78A25.
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1. Introduction

When we seek to localize an inhomogeneity of small volume contained in a three-dimensional bounded domain from a finite number of boundary measurements, we are usually concerned with an underlying inverse problem which is not in general well-posed. In the situation where the inverse problem is based on linear equations, the combination of an asymptotic expansion of the perturbation in the physical field in presence in the domain, with a suited inversion algorithm, can allow one to overcome the ill-posed character of this inverse problem. This is the approach proposed by Cedio-Fengya, Moskow & Vogelius [10] for localizing a finite number of conductivity inhomogeneities, of small volume, contained in a bounded domain. Typically, the inversion algorithm makes use of an asymptotic expansion for perturbations in the voltage potential, and is based on a minimization process of least-squares type for the calculation of the geometrical parameters of the inhomogeneities. The resulting localization procedure is therefore iterative, in contrast to the procedure developed by Ammari, Moskow & Vogelius [4]

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for such inhomogeneities. In fact, although using the same asymptotic expansion for measuring boundary voltage perturbations initiated by boundary electric currents, the inversion algorithm in [4] is based on one of the following two direct inversion processes: the Current Projection method and an Inverse Fourier method. In the first case, the algorithm consists of identifying the 'center' of a single inhomogeneity as the unknown of a linear system whereas in the second case, in the presence of multiple inhomogeneities, it consists of calculating a discrete inverse Fourier transform of a sample of measurements. Such direct processes appear numerically efficient (see e.g. [4, 22]) for solving inverse problems where we are mainly interested in the position of the single inhomogeneity, or in the positions of a finite number of inhomogeneities, in the domain. We will be concerned with such processes in this paper. For other numerical methods that could be used in the localization problem of conductivity inhomogeneities, or of dielectric inhomogeneities, in different settings, we refer to [2, 8, 12, 15, 16, 20, 21].

Recently, a framework for the localization of three-dimensional electromagnetic inhomogeneities was introduced by Ammari, Vogelius & Volkov [5]. This framework considers the time-harmonic Maxwell equations in a three-dimensional bounded domain Ω containing a finite number m of unknown inhomogeneities of small volume, and proposes to localize these inhomogeneities from an asymptotic expansion devoted to the study of perturbations in the tangential boundary trace of the curl of the electric field. A particular reformulation of this asymptotic expansion leads to an asymptotic formula that allows one to evaluate boundary measurements of "voltage" type from prescribed boundary currents. In the presence of well-separated inhomogeneities, and also distant from $\partial\Omega$, the boundary of Ω, this asymptotic formula states that:

$$\begin{aligned}
&\int_{\partial\Omega} \operatorname{curl} E_\alpha \times \nu \cdot w \, d\sigma - \int_{\partial\Omega} \operatorname{curl} w \times \nu \cdot (\nu \times (E_\alpha \times \nu)) \, d\sigma \\
&= \alpha^3 \sum_{j=1}^{m} \omega^2 \varepsilon_0 \mu_0 (\frac{\varepsilon_0}{\varepsilon_j} - 1) \left[M^j (\frac{\varepsilon_0}{\varepsilon_j}) E_0(z_j) \right] \cdot w(z_j) \\
&\quad + \alpha^3 \sum_{j=1}^{m} (\frac{\mu_0}{\mu_j} - 1) \left[M^j (\frac{\mu_0}{\mu_j}) \operatorname{curl} E_0(z_j) \right] \cdot \operatorname{curl} w(z_j) + \mathcal{O}(\alpha^4), \qquad (1.1)
\end{aligned}$$

where w denotes any smooth vector-valued function such that

$$\operatorname{curl}(\operatorname{curl} w) - k^2 w \,=\, 0 \text{ in } W,$$

with W an open neighborhood of Ω, $k^2 = \omega^2 \varepsilon_0 \mu_0$, and ω a given frequency. In (1.1), α is the common order of magnitude of the diameters of the inhomogeneities, and the points z_j, $1 \leq j \leq m$, represent the 'centers' of the inhomogeneities. The electric field is denoted by E_α in the presence of the inhomogeneities and by E_0 in the absence of all the inhomogeneities. The outward unit normal to Ω, defined on $\partial\Omega$, is represented by ν. The (constant) background magnetic permeability and complex permittivity are μ_0 and ε_0 respectively. Also, μ_j and ε_j are the (constant) magnetic permeability and the complex permittivity of the jth inhomogeneity. Finally, $M^j(\mu_0/\mu_j)$ and $M^j(\varepsilon_0/\varepsilon_j)$ are the polarization tensors associated with the jth inhomogeneity (symmetric 3×3 matrices).

More recently, Asch & Mefire [7] have achieved in various contexts the numerical localization of such electromagnetic inhomogeneities from three numerical procedures based on (1.1). Typically, each of these procedures results from the combination of (1.1) with one of the following inversion processes: the Current Projection method, an Inverse Fourier method, and a MUSIC