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A FINITE ELEMENT METHOD WITH RECTANGULAR PERFECTLY MATCHED LAYERS FOR THE SCATTERING FROM CAVITIES*

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Abstract

We develop a finite element method with rectangular perfectly matched layers (PMLs) for the wave scattering from two-dimensional cavities. The unbounded computational domain is truncated to a bounded one by using of a rectangular perfectly matched layer at the open aperture. The PML parameters such as the thickness of the layer and the fictitious medium property are determined through sharp a posteriori error estimates. Numerical experiments are carried out to illustrate the competitive behavior of the proposed method.

Mathematics subject classification: 35Q60, 65L60, 78A45.
Key words: Cavity, Perfectly matched layers, Finite element method.

1. Introduction

Consider a time-harmonic electromagnetic plane wave incident on a shaped open cavity embedded in an infinite ground plane. The ground plane and the walls of the open cavity are perfect electric conductors (PEC), and the interior of the open cavity is filled with a non-magnetic inhomogeneous material. The half-space above the ground plane is filled with a homogeneous, linear, isotropic medium characterized by its permittivity ε_0 and permeability μ_0. In the TM and TE polarization, we study the diffraction problem by a finite element method with rectangular perfectly matched absorbing layers. Several computational experiments indicate that the method is efficient.

The study of the wave scattering by a 2-D cavity-backed aperture in the infinite ground plane has been of great importance in aircraft industries. There has been a considerable interest in computation and design of cavities, see, e.g., [4, 12, 16]. However, there has not been much studied on the analysis of the problem. Recently, in [1, 2], Ammari and Bao developed a variational approach for solving the cavity problems in two- and three-dimensional media, and studied the well-posedness of the problem. They also investigated the problem by an integral equation method in [3]. In [18], we introduced a perfectly matched layer in curvilinear coordinates to study the locally perturbed half plane problems (including the cavity problems), and presented several numerical results.

The purpose of this paper is to develop efficient numerical methods for solving the cavity scattering problems. The main difficulty is to truncate the infinite domain into a bounded computational domain. The method studied in [1, 2] is based on a variational formulation in the cavity with a transparent boundary condition at the open aperture. The boundary operators are nonlocal, which yields some difficulties in practical computations. In [18], we overcome the

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difficulty by introducing the PML in curvilinear coordinates. However, for wide open cavities it will lead to large computational costs.

The purpose of this paper is twofold: First we explore the possibility of introducing a rectangular perfectly matched layer to deal with the difficulty in truncating the unbounded domain. Second we explore the possibility of using an error analysis to determine the PML parameters such as the thickness of the PML region and the medium property inside the region. We hope the ideas developed in this paper will be useful for solving other locally perturbed half plane problems.

The basic idea of the PML technique is to surround the computational domain by a finite thickness layer of the specially designed model medium that would attenuate outgoing waves propagating from the computational domain. Since Berenger proposed the PML method for the time dependent Maxwell equations in [6], various constructions of PML absorbing layers have been proposed and studied in the literature. In [8], for the wave scattering by bounded obstacles, Collino and Monk derived the perfectly matched layer in curvilinear coordinates. Subsequently, Chen and Liu [7] established the convergence theory of the PML method to the solution of the original problem. We refer to Turkel and Yefet [13] for a review on various proposed models, and Lassas and Somersalo [10] for some study of mathematical properties of the PML equations.

The layout of the paper is as follows. In the next section, we state the model problem and derive the variational formulations. The well-posedness of the variational problems is also studied. In Section 3, we introduce our PML formulations, and establish the existence, uniqueness and convergence of the PML formulations. In Section 4, we present several numerical examples to illustrate the competitive behavior of the method.

2. Two-dimensional Cavity Problem

Consider a two-dimensional cavity D of arbitrary cross section embedded in a perfectly conducting medium (see Fig 2.1). Above the line $\{x_2 = 0\}$, the medium is homogeneous with a positive dielectric coefficient ε_0. The medium inside D is inhomogeneous with dielectric coefficient $\varepsilon(x_1, x_2)$. We assume that $\mathrm{Re}\,\varepsilon(x_1, x_2) > 0$ and $\mathrm{Im}\,\varepsilon(x_1, x_2) \geq 0$. In this paper, the media are assumed to be non-magnetic, and the magnetic permeability μ_0 is constant. We are interested in the scattering of an incident plane wave by the cavity.

We denote by Γ the cavity aperture, and S the cavity walls. Let $\mathrm{R}^2_+ = \{x \in \mathrm{R}^2 : x = (x_1, x_2),\ x_2 > 0\}$ be the region above the ground plane, and $\Gamma^c = \partial\mathrm{R}^2_+ \backslash \Gamma$. Let n be the unit outward normal to ∂D. Denote in the whole space

$$k^2(x) = \begin{cases} \omega^2 \varepsilon_0 \mu_0 & \text{in } \mathrm{R}^2_+, \\ \omega^2 \varepsilon(x) \mu_0 & \text{in } D. \end{cases}$$

With the perfectly electric conducting boundary condition in mind, we investigate the TM and TE cases separately.

2.1. TM Polarization

In this case, the incident electric field and the total electric field are parallel to the invariant dimension, i.e., $E_I = (0, 0, u^i)$ and $E = (0, 0, u)$. By the field continuity conditions, u vanishes