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SEPARATION OF SCALES IN ELASTICITY IMAGING: A NUMERICAL STUDY*

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Abstract

In magnetic resonance elastography, one seeks to reconstruct the shear modulus from measurements of the displacement field in the whole body. In this paper, we present an optimization approach which solves the problem by simply minimizing a discrepancy functional. In order to recover a complex anomaly in a homogenous medium, we first observe that the information contained in the wavefield should be decomposed into two parts, a "near-field" part in the region around the anomaly and a "far-field" part in the region away from the anomaly. As will be justified both theoretically and numerically, separating these scales provides a local and precise reconstruction.

Mathematics subject classification: 35R30,74L15,92C55.
Key words: Elastography, multi-scale imaging, anomaly reconstruction.

1. Introduction

Extensive work has been carried out in the past decade to image the elastic properties of human soft tissues by inducing motion. This broad field, called elasticity imaging or elastography, is based on the initial idea that shear elasticity can be correlated with the pathology of tissues.

There are several techniques that can be classified according to the type of mechanical excitation chosen (static compression, monochromatic, or transient vibration) and the way these excitations are generated (externally or internally). Different imaging modalities can be used to estimate the resulting tissue displacements.

Magnetic resonance elastography (MRE) is a new way of applying the idea of elastography. It can directly visualize and quantitatively measure the displacement field in tissues subject to harmonic mechanical excitation at low-frequencies.

The principle of the MRE relies on three steps: first applying dynamic shear to a tissue, then measuring displacement, and finally solving an inverse problem to get a map of Young's modulus. Each one of these three steps is a technical challenge by itself. The works [2, 9, 18] describe various frameworks of this problem. We will focus on the so called "steady state" elastography, whose principles have been described in [16], and more recently in [18]. A harmonic excitation is applied by vibrating a piezoelectric transducer onto the body. This vibration

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propagates in the tissue and thus produces a harmonic displacement field. Figure 1.1 shows the modulus of the complex amplitude of such a displacement field obtained experimentally. The displacement field is imaged in the whole volume by a specific sequence of Magnetic Resonance Imaging (MRI). Such a sequence is called an elastography sequence. Once this displacement is acquired, an inverse problem is used to reconstruct the shear modulus. Being able to image the displacement in the whole volume is a crucial feature of the technique because it gives much more detailed information than what can be obtained from boundary measurements. As we will explain in this paper, such an imaging system has better resolution.

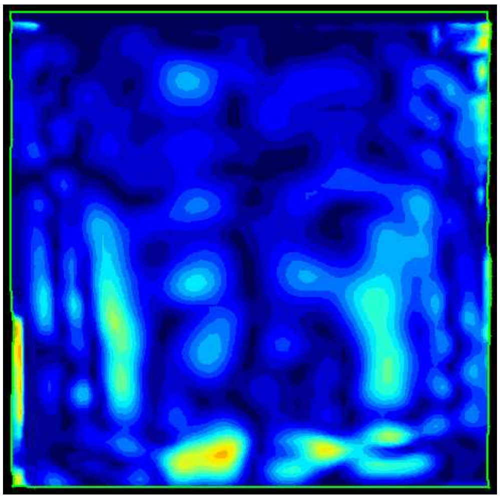

Fig. 1.1. Experimental wavefield in a phantom. The displacement field is acquired by an MRI sequence (R. Sinkus's group – LOA). The transducer is at the bottom. Although the waves are visible, the amplitude decays in the vertical direction because of a viscosity in the tissue.

We want to numerically estimate the local stiffness of the medium, given the displacement field produced by a known excitation. In this paper, we discuss the resolution performance one could expect when solving this kind of inverse problems with interior measurements.

The inverse problem for elastography is investigated in many different ways depending on the experimental setup. In MRE, it is common to solve the inverse problem by estimating the derivatives of the displacement field using finite different schemes; see for instance [17] and Section 2. Other contributors suggested to use finite element methods to match the displacement field; see, e.g., [20] and [19]. Our approach is somehow similar to this paradigm. We propose here a mathematical interpretation of the results. Indeed, our numerical simulation tool is different.

The paper is organized as follows. In Section 2, we recall our mathematical model for steady state elastography, and describe our optimization approach to solve the inverse problem. In Section 3, we investigate the resolution limits of our approach by considering a specific set of discontinuous coefficients to be recovered. In Section 4, we summarize the advantages and the disadvantages of our method by considering issues of stability, accuracy and computational performance.