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ADAPTIVE FINITE ELEMENT APPROXIMATION FOR A CLASS OF PARAMETER ESTIMATION PROBLEMS*

Karl Kunisch
Department of Mathematics, University of Graz, Austria
Email: karl.kunisch@uni-graz.at
Wenbin Liu
KBS, University of Kent, Canterbury, CT2 7NF, England
Email: w.b.liu@kent.ac.uk
Yanzhen Chang and Ningning Yan
LSEC, Academy of Mathematics and Systems Science,
Chinese Academy of Sciences, Beijing, China
Email: ynn@amss.ac.cn
Ruo Li
Department of Computational Mathematics, Peking University, China
Email: rli@math.pku.edu.cn

Abstract

In this paper, we study adaptive finite element discretisation schemes for a class of parameter estimation problem. We propose to use adaptive multi-meshes in developing efficient algorithms for the estimation problem. We derive equivalent a posteriori error estimators for both the state and the control approximation, which particularly suit an adaptive multi-mesh finite element scheme. The error estimators are then implemented and tested with promising numerical results.

Mathematics subject classification: 49J20, 65N30.
Key words: Parameter estimation, Finite element approximation, Adaptive finite element methods, A posteriori error estimate.

1. Introduction

Adaptive finite element approximation is very important in improving accuracy and efficiency of the finite element discretisation because it ensures a higher density of nodes in certain area of the computational domain, where the solution is more difficult to approximate. By now the theory and application of adaptive finite element methods for the numerical solutions of partial differential equations (PDEs)have reached some state of maturity as documented by a series of monographs. There has been so extensive research on developing adaptive finite element algorithms for PDEs in the scientific literature that it is simply impossible to give even a very brief review here.

Recently, there has been intensive research in adaptive finite element method for optimal control problems, see, e.g., [2–4,16,19–22]. The main existing approaches are the goal-orientated a posteriori error estimators, see, e.g., [3,4], and the residual based a posteriori error estimators, see, e.g., [16, 19, 20], where a posteriori error estimates equivalent to the energy norm of the

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approximation error were derived for several types of optimal control problems. The readers can refer to the recent monograph [21] for more details.

Furthermore, it has been found that for constrained control problems, different adaptive meshes are often needed for the control and the states, see [15, 16]. Using different adaptive meshes for the control and the state allows very coarse meshes to be used in solving the state and co-state equations. Thus much computational work can be saved since one of the major computational loads in computing optimal control is to solve the state and co-state equations repeatedly. This will be also seen from our numerical experiments in Section 6.

In this paper, we are interested in the least-square formulation of the following parameter estimation problem in R^n $(n \leq 3)$:

$$\min_{u \in K} \{g(y) + j(u)\}, \tag{1.1}$$

subject to

$$-\text{div}(A\nabla y(u)) + uy(u) = f \quad \text{in } \Omega. \tag{1.2}$$

where u is defined on Ω, and Ω is a bounded and simply connected open sets in R^n $(n \leq 3)$ with Lipschitz continuous boundaries $\partial\Omega$. Here $j(u) = \int_\Omega h(u)$ is convex functional, $f \in L^2(\Omega)$, and K is a closed convex set. We also assume that g and j are convex functionals which are continuously differentiable, and j is further strictly convex with $j(u) \to +\infty$ as $\|u\|_U \to \infty$, $g(\cdot)$ is bounded below. For the matrix A we assume that $A(\cdot) = (a_{i,j}(\cdot))_{n\times n} \in (W^{1,\infty}(\Omega))^{n\times n}$, such that there is a constant $c > 0$ satisfying that for any vector $X \in R^n$, it gives $X^tAX \geq c\|X\|^2_{R^n}$. The above problem is of course a class of optimal control problem. In comparison with the standard optimal control problems, there were relatively fewer known results in developing adaptive finite element approximation for parameter estimation problems due to the lower regularity of the parameter that often is discontinuous. In [5], goal-orientated a posteriori error estimators were developed for a class of parameter identification problem, and computational tests were presented. In [7, 8, 13], a posteriori error estimators of residual type were developed for the same problem but with stronger assumptions on the estimated parameter as required by the techniques used. In particular, these assumptions eliminate any jumps in the estimated parameter. Very recently a priori error estimates and super-convergence were presented in [26] for the above estimation problem, although much more work in convexity of the functional, regularity of the parameter, and a posteriori error estimation techniques was still needed before a posteriori error estimators of residual type can be rigorously derived.

The purpose of this work is to develop residual a posteriori error estimators for the adaptive finite element approximation of the above problem. In our work, the estimated parameter is assumed just in L^2 so that jumps in value are allowed for the estimated parameter. The plan of the paper is as follows. In Section 2, we introduce some notations and preliminaries. In Section 3, we will construct the finite element approximation for the parameter estimation problem. In Sections 4 and 5, sharp a posteriori error estimators are derived for the parameter identification problem. Finally numerical test results are presented in Section 6.

2. Notations and Preliminaries

2.1. Some notations

We adopt the standard notation $W^{m,q}(\Omega)$ for Sobolev spaces on Ω with norm $\|\cdot\|_{W^{m,q}(\Omega)}$ and seminorm $|\cdot|_{W^{m,q}(\Omega)}$ (or $\|\cdot\|_{m,q,\Omega}$, $|\cdot|_{m,q,\Omega}$ for simplification). Further we set $W_0^{1,q}(\Omega) \equiv$