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A NUMERICAL STUDY FOR THE PERFORMANCE OF THE WENO SCHEMES BASED ON DIFFERENT NUMERICAL FLUXES FOR THE SHALLOW WATER EQUATIONS *

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Abstract

In this paper we investigate the performance of the weighted essential non-oscillatory (WENO) methods based on different numerical fluxes, with the objective of obtaining better performance for the shallow water equations by choosing suitable numerical fluxes. We consider six numerical fluxes, i.e., Lax-Friedrichs, local Lax-Friedrichs, Engquist-Osher, Harten-Lax-van Leer, HLLC and the first-order centered fluxes, with the WENO finite volume method and TVD Runge-Kutta time discretization for the shallow water equations. The detailed numerical study is performed for both one-dimensional and two-dimensional shallow water equations by addressing the issues of CPU cost, accuracy, non-oscillatory property, and resolution of discontinuities.

Mathematics subject classification: 65M60, 65M99, 35L65.
Key words: Numerical flux, WENO finite volume scheme, Shallow water equations, High order accuracy, Approximate Riemann solver, Runge-Kutta time discretization.

1. Introduction

In this paper, we investigate the performance of the WENO methods based on different numerical fluxes for the shallow water equations, with the objective of obtaining better performance by choosing suitable numerical fluxes. The weighted essential non-oscillatory (WENO) scheme [10,14] is a procedure of spatial discretization; namely, it is a procedure to approximate the spatial derivative terms. The WENO scheme uses the idea of adaptive stencils in the reconstruction procedure based on the local smoothness of the numerical solution to automatically achieve high order accuracy and a non-oscillatory property near discontinuities. The WENO method has been developed in recent years as a class of high order method for the shallow water equations [3, 5, 26, 27], which gives sharp, non-oscillatory discontinuity transitions and at the same time provides high order accurate resolutions for the smooth part of the solution. The WENO schemes are widely studied after the structure of the finite difference WENO schemes were proposed from the ENO schemes by Jiang and Shu in 1996 [10]. The construction of finite

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volume WENO methods on unstructured meshes was presented by Friedrichs [6]. Instead of constructing two-dimensional finite volume WENO schemes with dimensional by dimensional methods, Hu and Shu [9] proposed a full dimensional reconstruction methodology for the third order WENO schemes by using a combination of two-dimensional linear polynomials and the third and fourth order WENO schemes by using a combination of two-dimensional quadratic polynomials. Then the WENO schemes were used in the shallow water flows simulations. Xing and Shu [27] developed a treatment for the bed slope source term of the shallow water equations using the fifth-order finite difference WENO scheme. In [26], Vukovic and Sopta used the finite difference ENO and WENO to solve the one-dimensional shallow water flows in order to maintain genuine high-order accuracy. Both finite volume WENO and central WENO schemes were used in [5] to solve the shallow water equations; high accuracy and well balancing are obtained in the paper.

An important component of the WENO methods for the shallow water equations is the numerical flux based on exact or approximate Riemann solvers. In most of the WENO methods, Lax-Friedrichs (LF) numerical flux is used due to its simplicity. However, there exist many other numerical fluxes based on exact or approximate Riemann solvers documented in the book of Toro [23, 24], which could also be used in the context of the WENO method. The high-order accurate WENO schemes and the HLLC approximate Riemann solver are used in compressible multicomponent flow problems [11]. A comparison is made between the difference schemes of Engquist, Fisher, and Roe fluxes with Galerkin methods for approximating hyperbolic conservation laws [16]. The Godunov flux [7, 23, 24] is based on exact Riemann solver, which has the smallest viscosity among all the monotone numerical fluxes, but it often lacks explicit formulas and relies on iterative procedures. In this paper, we will consider six numerical fluxes except the Godunov flux based on approximate Riemann solvers, which are LF flux, local LF (LLF) flux, EO flux, Harten-Lax-van Leer (HLL) flux, HLLC flux, the first-order centered (Force) flux, and compare the performance of the WENO methods based on these numerical fluxes for the shallow water equations, with the objective of obtaining better performance by choosing suitable numerical fluxes. We review and describe the details of the numerical fluxes under consideration in section 2, and present extensive numerical tests in section 3 to compare their performance for the shallow water equations. The detailed numerical study is performed for the one-dimensional and two-dimensional shallow water equations. Concluding remarks are given in section 4.

2. Review and Implementation of the Numerical Fluxes for the WENO Method

In this section, we review the WENO method for the shallow water equations and the numerical fluxes under consideration of the WENO method. We mainly describe the WENO method for the one-dimensional case. Consider the one-dimensional shallow water equations:

$$U_t + F(U)_x = S(U), \tag{2.1}$$

with

$$U = [h, hu]^T, \;\; F(U) = [hu, hu^2 + \frac{1}{2}gh^2]^T, \;\; S = [0, -ghb_x]^T,$$

where U is the vector of conservative variables, F is the flux vector, S is the source term relative to the bottom slope, t is the time, x is the space, h is the water height, u is the vertically averaged velocity, g is the gravity, and b is the bottom elevation.