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NUMERICAL SOLUTION OF THE SCATTERING PROBLEM FOR ACOUSTIC WAVES BY A TWO-SIDED CRACK IN 2-DIMENSIONAL SPACE*

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Abstract

The wave scattering problem by a crack Γ in $\mathbb{R}^2$ with impedance type boundary is considered. This problem models the diffraction of waves by thin two-sided cylindrical screens. A numerical method for solving the problem is developed. The solution of the problem is represented in the form of the combined angular potential and single-layer potential. The linear integral equations satisfied by the density functions are derived for general parameterized arcs. The weakly singular integrals and the Cauchy singular integral arising in these equations are computed using a highly accurate scheme with a truncation error analysis. The advantage of the scheme proposed in this paper is, in one hand, the fact that we do not need the analyticity property of the crack and we allow different complex valued surface impedances in both sides of the crack. In the other hand, we avoid the hyper-singular integrals. Numerical implementations showing the validity of the scheme are presented.

Mathematics subject classification: 35P25, 35R30, 78A45.
Key words: Wave scattering, Impedance boundary, Integral equations, Singularity analysis, Numerics.

1. Introduction

The scattering problems for acoustic waves in $\mathbb{R}^2$ are of great importance, which can be considered as a model problem of cylinder scattering in $\mathbb{R}^3$. Mathematically, such problems are governed by the Helmholtz equations in $\mathbb{R}^2$ with boundary value specified on the boundary of the scatterer and the radiation condition at infinity for scattered wave. In case the scatterer D is a body with a closed smooth surface, these scattering problems have been extensively studied using the potential methods [4,9,25,28,32], the scattering problem for multiple obstacle is also considered in [31]. In these cases, the combined single- and double-layer potential schemes [4, 9, 28] are proposed to express the scattered wave as well as its far-field pattern by density functions, which satisfies an integral equation derived from the boundary condition in

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∂D. However, to guarantee the solvability for the density functions in the combined single– and double–layer potential scheme, the hyper-singular integrals need to be computed numerically for ∂D with Neumann or Robin boundary condition [3, 23, 24].

To describe the diffraction of waves by thin two-side cylindrical screens, the scattering problems of Helmholtz equation by a crack Γ (or, an open arc) in $\mathbb{R}^2$ are considered recently. In this case, the boundary condition on the arc should be specified, while the acoustic property of the arc in both sides may be different. For arc scattering problems, the scattered wave is continuous at the tips, but its gradient has weak power singularity. Consequently, the potential functions introduced for solving the scattering problems are not periodic nor smooth in the closure of Γ any more. Hence, compared with the scattering problems of obstacles with closed smooth boundary, these crack scattering problems are more complicated due to the eventual different surface impedances on both sides as well as the presence of the tips of crack. The inverse scattering problems for arc with Dirichlet or Neumann data on the arc has been considered in [12, 14, 17, 18, 22, 30, 33, 34] using iterative methods. Physically, this means that the arc is sound-soft or sound hard. For the Lapalce equation in $\mathbb{R}^2$ with a cut inside the domain, the Cauchy problem is also considered in [6], where both the Dirichlet and the Neumann condition are posed in both sides of the cut in the iteration process.

Let us emphasize that the impedance condition specified on the obstacle boundary is of great importance in application [10]. The introduction of surface impedance specified on the boundary has a big influence on the scattered wave. For example, an inverse scattering problem for an impenetrable obstacle with smooth surface has been studied in [5, 26, 27], where the impedance coefficients on the boundary are found to have a strong influence on the scattering process. This is related to the so-called coating effect which, in few words, means that these coefficients can increase or decrease the amount of scattering. In [21], an inverse scattering problem for an arc with different impedance coefficients at two sides of the arc is studied using a direct method, where the direct scattering process is simulated by the combined single- and double-layer potential method.

In this paper, we develop a numerical method for the direct scattering problem of a two-sided impedance arc $\Gamma \in \mathbb{R}^2$. By two-sided arc, we mean that the acoustic property of the arc in two sides may be different. If we specify two sides of Γ by Γ^+ and Γ^-, then the boundary impedance coefficient as well as the boundary data in Γ^+ and Γ^- may be different. The numerical method developed in the present paper is based on the boundary integral equation approach proposed in [15], where the integral representation for a solution of the problem is obtained in the form of a sum of a single layer potential and an angular potential. In the case of an arc with the same impedance condition on both sides, the computational scheme has been given in [11, 13], where the analytic property of the arc and the hyper-singular integral are required. But their approach seems not clear how to be applied for different impedances. On the other hand, we would like to point out the recent developments in connection with our studied problem and suggested techniques, specially for small cracks, see, e.g., [1, 2].

The purpose of this paper is to give an efficient realization scheme for computing the scattered wave caused by a two-sided complex arc, using the combined angular- and single-layer potential method. Although the integral equation for the density functions has been derived in [15] and its solvability is proven there, the efficient numerical realization of this scheme is still open. More precisely, the integral system consists of a Cauchy singular integral equation of the 1st kind with additional integral condition and an integral equation of the second kind with smooth kernels. Since the Cauchy singular operator has additional weak singularities at