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A PREDICTOR MODIFICATION TO THE EBDF METHOD FOR STIFF SYSTEMS*

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Abstract

In this paper we modify the EBDF method using the NDFs as predictors instead of BDFs. This modification, that we call ENDF, implies the local truncation error being smaller than in the EBDF method without losing too much stability. We will also introduce two more changes, called ENBDF and EBNDF methods. In the first one, the NDF method is used as the first predictor and the BDF as the second predictor. In the EBNDF, the BDF is the first predictor and the NDF is the second one. In both modifications the local truncation error is smaller than in the EBDF. Moreover, the EBNDF method has a larger stability region than the EBDF.

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Key words: Backward differentiation formula (BDF), EBDF, Predictor, Stability, Stiff Systems.

1. Introduction

We will consider the following initial value problem (IVP):

$$y'(x) = f\left(x, y(x)\right), \quad y\left(x_0\right) = y_0 \tag{1.1}$$

where $T = [x_0, x_n]$ is a finite interval and y: $[x_0, x_n] \to \mathbb{R}^m$ and f: $[x_0, x_n] \times \mathbb{R}^m \to \mathbb{R}^m$ are continuous functions.

When we are working with a stiff problem, the numerical method used must be accurate and it needs an extensive stability region too [4]. Because of the latter reason, in the recent years many researches have been focused on developing convenient numerical methods for stiff problems and a lot of improvements have been made on the basis of the backward differentiation formula (BDF) introduced by Gear [6], due to its good stability properties.

One of the modifications done to the BDFs are the NDFs (Numerical Differentiation formulae). It is a computationally cheap modification that consists of anticipating a difference of order $(k+1)$ multiplied by a constant $\kappa\gamma_k$ in the BDF formula of order k. This term has a positive effect on the local truncation error, making the NDFs more accurate than the BDFs and not much less stable. This modification was proposed by Shampine [10] but only for orders $k = 1, 2, 3, 4$, because it is inefficient for orders greater than 4.

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In [1] and [2] Cash introduces methods using superfuture points to solve stiff IVPs. These methods are known as extended BDF (EBDF) and modified extended BDF (MEBDF). They consist of applying the BDF predictors twice and one implicit multistep corrector. Both methods use superfuture points to gain stability and they are A-stable up to order 4 and $A(\alpha)$-stable up to order 9. In [3] a code based on the MEBDF is described and in [8] Matrix free MEBDF (MF-MEBDF) methods are introduced to optimize the computations of the EBDF. A different variation of the BDFs was introduced by Fredebeul [5], the A-BDF method. In this method the implicit and explicit BDF are used in the same formula, with a free parameter, being $A(\alpha)$-stable up to order 7.

In this paper, we follow the EBDF scheme but substituting the BDF predictors by the NDF formulae. In the ENDF method we will use the NDFs as predictors maintaining the last corrector of the EBDF. The result of this application will be a smaller local truncation error and a not too much smaller stability region than in the EBDF. Next, we introduce two modifications more, the EBNDF and ENBDF maintaining the corrector of the EBDF scheme. In EBNDF the first predictor is the BDF and the second one the NDF. In ENBDF, the first predictor is the NDF and the second BDF. Both of them have a smaller local truncation error than EBDF, and in the case of EBNDF also the stability region is bigger than the one of the EBDF.

The article is organised as follows: in Section 2 we give details about modifications introduced in EBDF, such as ENDF, ENBDF and EBNDF. In Section 3 the stability analysis is developed and we include some computational aspects as well as numerical examples of ODEs with different stiffness ratios in Section 4.

2. Using NDFs as Predictors in the EBDF Scheme

In this Section we will start analysing the properties of the NDF and EBDF and finally we will derive the ENDF, ENBDF and EBNDF algorithms.

2.1. NDF scheme

Since they were introduced by Gear [6], the Backward differentiation formulae have been widely used due to their good stability properties for solving stiff problems. The BDF of order k can be expressed as follows:

$$\sum_{j=1}^{k} \frac{1}{j} \nabla^j y_{n+k} = h f_{n+k}. \tag{2.1}$$

Developing the backward differences of expression (2.1) we get the well-known expression for the BDF:

$$\sum_{j=0}^{k} \hat{\alpha}_j y_{n+j} = h f_{n+k}. \tag{2.2}$$

The local truncation error (LTE) of the BDF of order k is given by the following expression

$$LTE_k = C_1 h^{k+1} y^{(k+1)}(x_n) + \mathcal{O}\left(h^{k+2}\right), \tag{2.3}$$