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A FAST HIGH ORDER METHOD FOR ELECTROMAGNETIC SCATTERING BY LARGE OPEN CAVITIES*

Meiling Zhao
School of Mathematics and System Science, Beijing University of Aeronautics & Astronautics, Beijing 100191, China
Email: meilingzhaocn@yahoo.com
Zhonghua Qiao*
Institute for Computational Mathematics & Department of Mathematics, Hong Kong Baptist University, Kowloon, Hong Kong
Email: zqiao@hkbu.edu.hk
Tao Tang
Department of Mathematics, Hong Kong Baptist University, Kowloon, Hong Kong
Email: ttang@hkbu.edu.hk

Abstract

In this paper, the electromagnetic scattering from a rectangular large open cavity embedded in an infinite ground plane is studied. By introducing a nonlocal artificial boundary condition, the scattering problem from the open cavity is reduced to a bounded domain problem. A compact fourth order finite difference scheme is then proposed to discrete the cavity scattering model in the rectangular domain, and a special treatment is enforced to approximate the boundary condition, which makes truncation errors reach $\mathcal{O}(h^4)$ in the whole computational domain. A fast algorithm, exploiting the discrete Fourier transformation in the horizontal and a Gaussian elimination in the vertical direction, is employed, which reduces the discrete system to a much smaller interface system. An effective preconditioner is presented for the BICGstab iterative solver to solve this interface system. Numerical results demonstrate the remarkable accuracy and efficiency of the proposed method. In particular, it can be used to solve the cavity model for the large wave number up to 600π.

Mathematics subject classification: 65N06, 78M20.
Key words: Electromagnetic cavity, Compact finite difference scheme, FFT, Preconditioning.

1. Introduction

The scattering properties of open cavities are of high interest to the engineering community, with a number of applications including the design of jet engine inlet ducts and cavity-backed antenna for military and civil use. In this paper we mainly concern with the electromagnetic scattering from a two-dimensional large open cavity as shown in Fig. 1.1. The ground plane and the walls of the open cavity are assumed as perfect electric conductors (PEC), and the interior of the open cavity is filled with non-magnetic materials which may be inhomogeneous. The half space above the ground plane is filled with a homogenous and isotropic medium with its permittivity ε_0 and permeability μ_0. In this setting, the electromagnetic scattering by the

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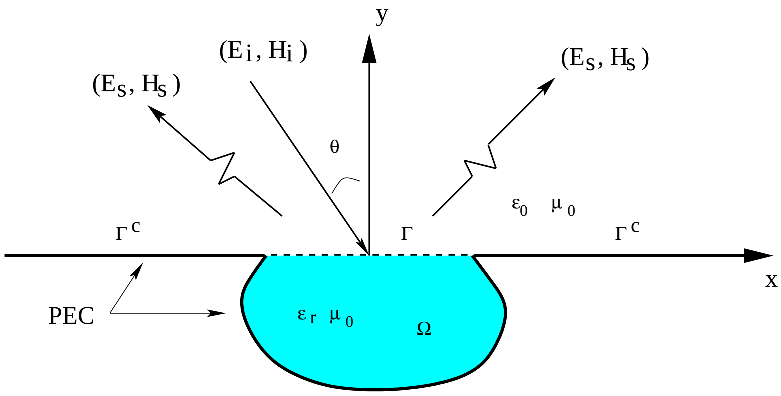

Fig. 1.1. The geometry of the cavity.

cavity is governed by the Helmholtz equation along with Sommerfeld's radiation conditions imposed at infinity. A variety of numerical methods, including the method of moments, finite difference, finite element, boundary element method, and hybrid methods [3, 7, 9, 10], have been developed to characterize the scattering from cavities. Some mathematical analysis and numerical treatments on the open cavities can be found in [1, 2, 5].

However, the problem will be challenging when the wave number k is large, or the cavity becomes large compared to the wavelength of the fields, more precisely large ka, where a denotes the size of the computational domain, because of the highly oscillatory nature of the fields. For a large wave number, the phase error (pollution) of the computed solution obtained with low order discretization is large unless fine meshes are used per wavelength. See [22] for detailed information. A fine mesh would lead to a large system of equations which may be computationally prohibitive. For instance, a large cavity, 1m ×1m, with a centimeter incident wave, this condition means when choosing mesh density as $1/20 \sim 1/40$ of the wavelength, it will produce 10^8 unknowns in the discrete linear systems of the two-dimensional case. Many numerical approaches have been proposed to reduce the phase error. For example, the high-order finite element method was proposed in [7]; the h-version and h-p-version finite element methods were proposed in [23, 24]. In [11], a standard bilinear finite element together with a modified quadrature rule was used, which led to fourth order phase accuracy on orthogonal uniform meshes. The high order spectral method and compact high order finite difference method have been presented to solve the Helmholtz equation in [12,13,16–18,25]. In [8], a fully high-order finite element with curvilinear tetrahedral elements was developed to simulate the scattering by cavities. High order methods are attractive for solving the Helmholtz problem with the large wave number since they can offer relative higher accurate solution by utilizing fewer mesh points and spending less computational costs than the low order approaches.

For the cavity electromagnetic scattering problem, the accurate computation for the radar cross section (RCS) is of particular importance. Bao and Sun proposed a fast algorithm in [3] for solving the electromagnetic scattering from a rectangular cavity. Using the discrete Fourier transform in the horizontal direction and a Gaussian elimination in the vertical direction, the approach reduces the global system in the entire cavity to an interface linear system on the top line of the cavity with computational complexity proportional to the number of the unknowns by appropriate iterative methods in the source free case. This algorithm was further improved in [14].

In this work, we propose a fast high order finite difference method for the scattering of