Journal of Computational Mathematics
Vol.29, No.6, 2011, 623–638.

http://www.global-sci.org/jcm
doi:10.4208/jcm.1110-m11si02

HIGH ORDER WEIGHTED ESSENTIALLY NON-OSCILLATION SCHEMES FOR ONE-DIMENSIONAL DETONATION WAVE SIMULATIONS*

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Abstract

In this paper, three versions of WENO schemes WENO-JS, WENO-M and WENO-Z are used for one-dimensional detonation wave simulations with fifth order characteristic based spatial flux reconstruction. Numerical schemes for solving the system of hyperbolic conversation laws using the ZND analytical solution as initial condition are presented. Numerical simulations of one-dimensional detonation wave for both stable and unstable cases are performed. In the stable case with overdrive factor $f = 1.8$, the temporal histories of peak pressure of the detonation front computed by WENO-JS and WENO-Z reach the theoretical steady state. In comparison, the temporal history of peak pressure computed by the WENO-M scheme fails to reach and oscillates around the theoretical steady state. In the unstable cases with overdrive factors $f = 1.6$ and $f = 1.3$, the results of all WENO schemes agree well with each other as the resolution, defined as the number of grid points per half-length of reaction zone, increases. Furthermore, for overdrive factor $f = 1.6$, the grid convergence study demonstrates that the high order WENO schemes converge faster than other existing lower order schemes such as unsplit scheme, Roe's solver with minmod limiter and Roe's solver with superbee limiter in reaching the predicted peak pressure. For overdrive factor $f = 1.3$, the temporal history of peak pressure shows an increasingly chaotic behavior even at high resolution. In the case of overdrive factor $f = 1.1$, in accordance with theoretical studies, an explosion occurs and different WENO schemes leading to this explosion appear at slightly different times.

Mathematics subject classification: 65P30, 77Axx.
Key words: Weighted Essentially Non-Oscillatory, Detonation, ZND.

1. Introduction

Detonation is a complex phenomenon that involves a shock front followed by a reaction zone. The classical theory in detonation waves was pioneered by Zekdivich [1], von Neumann [2] and Doering [3], namely the ZND detonation model. Then both theoretical studies and numerical techniques have been developed to investigate the detonation phenomenon in many physical

* Received January 26, 2011 / Revised version received August 5, 2011 / Accepted August 10, 2011 / Published online November 15, 2011 /

applications. Numerical approaches such as PPM with front tracking and mesh refinement [4], Roe's solver with superbee limiter and the minmod limiter [5], unsplit scheme [6], and WENO-M with shock fitting [7] have been implemented to simulate detonation waves to study its instability and mechanisms.

Although detonation has been studied for many years, it remains an active area of research in both theoretical studies and in numerical simulations due to the practical importance. In this paper, we are interested in the numerical simulations of one-dimensional detonation waves by Weighted Essentially Non-Oscillation (WENO) schemes, which have been developed in recent years as a class of high order/high resolution method for solutions of hyperbolic conservation laws in the presence of shocks and small scale structures in the solution.

The local computational stencils of $(2r-1)$ order WENO schemes are composed of r overlapping substencils of r points, forming a larger stencil with $(2r-1)$ points. The scheme yields a local rate of convergence that goes from order r at the non-smooth parts of the solution, to order $(2r-1)$ when the convex combination of local lower order polynomials is applied at smooth parts of the solution. The nonlinear coefficients of WENO's convex combination, hereafter referred to as *nonlinear weights* ω_k, are based on lower order local smoothness indicators $\beta_k, k = 0, \cdots, r-1$ that measure the sum of the normalized squares of the scaled L^2 norms of all derivatives of r local interpolating polynomials. An essentially zero weight is assigned to those lower order polynomials whose underlining substencils contain high gradients and/or shocks, aiming at an essentially non-oscillatory solution close to discontinuities. At smooth parts of the solution, the formal order of accuracy is achieved through the mimicking of the central upwind scheme of maximum order, when all smoothness indicators are about the same size. Hence, the one of the most important issues for WENO schemes is to design an efficient and accurate nonlinear weights ω_k. In [8], the first set of nonlinear weights of widespread use has been given. However, it has been shown that the nonlinear weights fail to satisfy the necessary and sufficient condition for achieving the formal order of accuracy even for smooth functions. We call this scheme as the classical WENO scheme (WENO-JS). In [9], a modification of the nonlinear weights was proposed in the form of a mapping on the classical WENO-JS nonlinear weights, leading to corrected nonlinear weights that recovered the formal order of accuracy. We call the scheme composed by this mapped set of nonlinear weights as the mapped WENO scheme (WENO-M). In [10], it was shown that the incorporation of a global optimal order smoothness indicator, hereafter denoted as τ_{2r-1}, into the classical WENO-JS nonlinear weights definition satisfies the necessary and sufficient condition for achieving the formal order of accuracy. This scheme has been named the WENO-Z scheme (WENO-Z). The mapping procedure of WENO-M incurs extra expensive computational cost, while the weight modification of WENO-Z is obtained through a simple and inexpensive linear combination of the already computed lower order local smoothness indicators β_k. It has been shown that the new set of nonlinear weights of WENO-Z provided less dissipation than WENO-JS and yielded comparable resolution of smooth solution and captured sharp gradients as WENO-M [10].

Numerical experiments showed that detonation waves for overdrive factor f, which is the square of the ratio of imposed detonation front velocity and the Chapman-Jouguet velocity, $f = 1.8$ is a stable case and lower overdrive factors $f = 1.6$, $f = 1.3$ and $f = 1.1$ are unstable cases. For the stable case, the temporal histories of peak pressure of the detonation front computed by WENO-JS and WENO-Z schemes reach the steady state while the temporal history of peak pressure computed by the WENO-M fails to reach the steady state. For the unstable cases, the results of three versions of WENO schemes agree well with each other as