Journal of Computational Mathematics
Vol.31, No.1, 2013, 57–67.

http://www.global-sci.org/jcm
doi:10.4208/jcm.1208-m4022

ON STRUCTURED VARIANTS OF MODIFIED HSS ITERATION METHODS FOR COMPLEX TOEPLITZ LINEAR SYSTEMS*

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Abstract

The *Modified Hermitian and skew-Hermitian splitting* (MHSS) iteration method was presented and studied by Bai, Benzi and Chen (Computing, 87(2010), 93-111) for solving a class of complex symmetric linear systems. In this paper, using the properties of Toeplitz matrix, we propose a class of structured MHSS iteration methods for solving the complex Toeplitz linear system. Theoretical analysis shows that the structured MHSS iteration method is unconditionally convergent to the exact solution. When the MHSS iteration method is used directly to complex symmetric Toeplitz linear systems, the computational costs can be considerately reduced by use of Toeplitz structure. Finally, numerical experiments show that the structured MHSS iteration method and the structured MHSS preconditioner are efficient for solving the complex Toeplitz linear system.

Mathematics subject classification: 65F10, 65F50.
Key words: Toeplitz matrix, MHSS iteration method, Complex symmetric linear system.

1. Introduction

We consider the complex Toeplitz linear system

$$Ax = b, \tag{1.1}$$

with $A \in \mathbb{C}^{n\times n}$ a large, non-Hermitian, and positive definite Toeplitz matrix, and $x, b \in \mathbb{C}^n$. The complex Toeplitz matrices arise in solutions of differential and integral equations, and in some practical problems and mathematical methods in physics, statistics, and signal processing.

* Received January 15, 2012 / Revised version received April 4, 2012 / Accepted August 7, 2012 / Published online January 17, 2013 /

A complex Toeplitz matrix A has the form

$$A = \begin{pmatrix} a_0 & a_1 & a_2 & \cdots & a_{n-1} \\ a_{-1} & \ddots & \ddots & \ddots & \vdots \\ a_{-2} & \ddots & \ddots & \ddots & a_2 \\ \vdots & \ddots & \ddots & \ddots & a_1 \\ a_{-n+1} & \ddots & a_{-2} & a_{-1} & a_0 \end{pmatrix}, \tag{1.2}$$

where $a_j \in \mathbb{C}$. From (1.2) it is easy to know that the matrix A is decided by the elements of its first row and first column. To solve the complex Toeplitz linear system (1.1), actual structure and efficient splitting of the Toeplitz matrix A are important and need to be discussed in depth.

Recently, the *Hermitian and skew-Hermitian splitting* (HSS) iteration method has been paid great attention. Bai, Golub and Ng introduced the HSS iteration method for solving the non-Hermitian positive definite linear systems in [8]. Based on the HSS iteration method, Bai, Benzi and Chen established the *modified HSS* (MHSS) iteration method for solving a class of important complex symmetric linear systems in [4], and proposed the *generalized MHSS* (GMHSS) iteration method, too. According to properties of the saddle point matrix, Bai and Golub proposed the *accelerated HSS* (AHSS) iteration method in [5]. In addition, Gu and Tian used the HSS iteration method to solve the Toeplitz linear system in [13], and Chen and Jiang presented the structured HSS and the structured AHSS iteration methods for solving the real Toeplitz linear systems in [11]; see also [1, 2, 6, 7, 9, 14, 15].

We now describe the HSS, MHSS and GMHSS iteration methods. To this end, the matrix A is split into its Hermitian and skew-Hermitian parts as

$$A = H + S,$$

where $H = \frac{1}{2}(A + A^*)$ and $S = \frac{1}{2}(A - A^*)$; see [3]. Here, A^* is used to denote the conjugate transpose of the matrix A. Based on this splitting, the HSS iteration method [8] can be described as follows.

The HSS Iteration Method. Given an initial guess $x^{(0)} \in \mathbb{C}^n$, for $k = 0, 1, 2, \ldots$ until the iteration sequence $\{x^{(k)}\}$ converges, compute $x^{(k+1)}$ using the following procedure:

$$\begin{cases} (\alpha I + H)x^{(k+\frac{1}{2})} = (\alpha I - S)x^{(k)} + b, \\ (\alpha I + S)x^{(k+1)} = (\alpha I - H)x^{(k+\frac{1}{2})} + b, \end{cases}$$

where α is a given positive constant, and I is the identity matrix.

When the coefficient matrix A is complex symmetric, we have

$$A = Re(A) + \imath Im(A),$$

where $Re(A)$ and $Im(A)$ denote the real and the imaginary parts of the matrix A, respectively, and $\imath$ represents the imaginary unit. For the complex symmetric linear system, the MHSS iteration method [4] is described as follows.

The MHSS Iteration Method. Given an initial guess $x^{(0)} \in \mathbb{C}^n$, for $k = 0, 1, 2, \ldots$ until the iteration sequence $\{x^{(k)}\}$ converges, compute $x^{(k+1)}$ using the following procedure:

$$\begin{cases} (\alpha I + Re(A))x^{(k+\frac{1}{2})} = (\alpha I - Im(A)\imath)x^{(k)} + b, \\ (\alpha I + Im(A))x^{(k+1)} = (\alpha I + Re(A)\imath)x^{(k+\frac{1}{2})} - \imath b, \end{cases}$$