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THE COUPLING OF NBEM AND FEM FOR QUASILINEAR PROBLEMS IN A BOUNDED OR UNBOUNDED DOMAIN WITH A CONCAVE ANGLE*

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Abstract

Based on the Kirchhoff transformation and the natural boundary element method, we investigate a coupled natural boundary element method and finite element method for quasi-linear problems in a bounded or unbounded domain with a concave angle. By the principle of the natural boundary reduction, we obtain natural integral equation on circular arc artificial boundaries, and get the coupled variational problem and its numerical method. Moreover, the convergence of approximate solutions and error estimates are obtained. Finally, some numerical examples are presented to show the feasibility of our method. Our work can be viewed as an extension of the existing work of H.D. Han et al..

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1. Introduction

The standard procedure of the coupling of boundary element and finite element can be described as follows. We introduce an artificial boundary to divide the original domain into two regions, an unbounded domain and a bounded one on which the boundary element method and finite element method are used, respectively.

In this paper, the coupling of natural boundary element method (NBEM) [4, 5, 19, 20] and finite element method (FEM) which is also called artificial boundary method [7–9] or DtN method [6, 13] is applied to solve boundary value problems in a bounded or unbounded domain with a concave angle.

Let Ω be a bounded and simple connected domain with sufficiently smooth boundary $\partial\Omega = \Gamma_0 \cup \Gamma_\alpha \cup \Gamma$, where

$$\Gamma_0 = \big\{(r,0) \mid 0 \le r \le a\big\},\ \Gamma_\alpha = \big\{(r,\alpha) \mid 0 \le r \le b\big\},$$
$$\Gamma = \big\{(r,\theta) \mid r = \psi(\theta) \ge 0,\ 0 \le \theta \le \alpha,\ \psi(0) = a, \psi(\alpha) = b\big\},$$

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and Γ is a smooth curve. Here α is a concave angle (Fig. 1.1). Particularly, when $\alpha = 2\pi$, Ω is a cracked domain. And Ω^c refers to the unbounded domain with boundary $\partial\Omega^c = \Gamma_0 \cup \Gamma_\alpha \cup \Gamma$, where Γ is defined as above and Γ_0 and Γ_α are changed by

$$\Gamma_0 = \{(r,0) \mid 0 \le a \le r\},\ \Gamma_\alpha = \{(r,\alpha) \mid 0 \le b \le r\}.$$

The problem can be described as follows [3,8,10,11,16].

$$\begin{cases} -\nabla\cdot(a(x,u)\nabla u) = f, & \text{in } \Omega \text{ or } \Omega^c, \\ \dfrac{\partial u}{\partial n} = 0, & \text{on } \Gamma_0 \cup \Gamma_\alpha, \\ u = 0, & \text{on } \Gamma, \end{cases} \tag{1.1}$$

where $a(\cdot,\cdot)$ and f are given functions with various properties which will be ranked in the following. When we study the domain Ω^c, problem (1.1) is not well posed. We need an appropriate boundary condition at infinity

$$u(x) \text{ is bounded, as } |x| \to \infty. \tag{1.2}$$

Problem (1.1)–(1.2) has many physical applications in, e.g., the field of magnetostatics, where a is the magnetic permeability and u is the magnetic scalar potential; the field of compressible flow, where a is the density and u is the velocity potential. See [2,15] etc. for more numerical results about problems of this kind with bounded domains. And note that, when we consider problem (1.1)-(1.2) in the unbounded domain Ω^c and get rid of the boundary conditions on Γ_0 and Γ_α, this is the right problem which was discussed in [8] by the artificial boundary method. Hence, our work can be viewed as a continuation of [8]. Moreover, we give an error analysis in Section 3, which was not presented in [8].

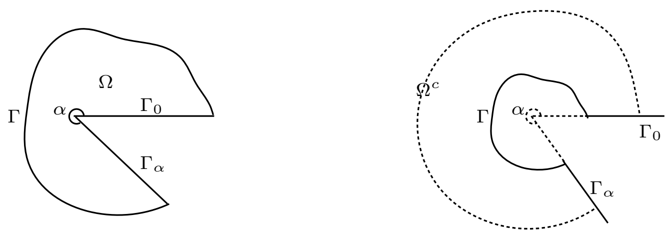

Fig. 1.1. The illustration of domains: the left is Ω, and the right is Ω^c.

Following [8,11], suppose that the given function $a(\cdot,\cdot)$ satisfies

$$0 < C_0 \le a(x,u) \le C_1,\ \forall\, u \in \mathbb{R}, \text{ and for almost all } x \in \Omega \text{ or } x \in \Omega^c, \tag{1.3}$$

where C_0, $C_1 \in \mathbb{R}$ are two constants, and

$$|a(x,u) - a(x,v)| \le C_L|u-v|,\ \forall\, u,v \in \mathbb{R} \text{ and for almost all } x \in \Omega \text{ or } x \in \Omega^c, \tag{1.4}$$

with a constant $C_L > 0$. We also assume that $\dfrac{\partial a}{\partial s}, \dfrac{\partial^2 a}{\partial s^2}$ are continuous.

In the following, we suppose that the function$f \in L^2(\Omega)$ or $f \in L^2(\Omega^c)$ has compact support, i.e., there exists a constant $R_0 > 0$, such that

$$\text{supp} f \subset \Omega_{R_0} = \Big\{x \in \mathbb{R}^2 \mid |x| \ge R_0\Big\} \text{ or } \Big\{x \in \mathbb{R}^2 \mid |x| \le R_0\Big\}, \tag{1.5}$$