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PARALLEL QUASI-CHEBYSHEV ACCELERATION TO NONOVERLAPPING MULTISPLITTING ITERATIVE METHODS BASED ON OPTIMIZATION*

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Abstract

In this paper, we present a parallel quasi-Chebyshev acceleration applied to the nonoverlapping multisplitting iterative method for the linear systems when the coefficient matrix is either an H-matrix or a symmetric positive definite matrix. First, m parallel iterations are implemented in m different processors. Second, based on l_1-norm or l_2-norm, the m optimization models are parallelly treated in m different processors. The convergence theories are established for the parallel quasi-Chebyshev accelerated method. Finally, the numerical examples show that the parallel quasi-Chebyshev technique can significantly accelerate the nonoverlapping multisplitting iterative method.

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1. Introduction and Preliminaries

A multisplitting of a nonsingular matrix $A \in R^{n\times n}$, as introduced in [12], is a collection of triples of $n \times n$ matrices $(M_i, N_i, E_i)_{i=1}^m (m \leq n$, a positive integer) with

- $A = M_i - N_i, \quad i = 1, \cdots, m;$
- M_i nonsingular, $i = 1, \cdots, m;$
- for $i = 1, \cdots, m$, the weighting matrices $E_i = diag(e_1^{(i)}, \cdots, e_n^{(i)})$ being diagonal with nonnegative entries

$$e_j^{(i)} = \begin{cases} e_j^{(i)} > 0, & \text{for } j \in \mathcal{N}_i, \\ 0, & \text{for } j \notin \mathcal{N}_i, \end{cases} \qquad j = 1, \cdots, n,$$

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such that $\sum_{i=1}^{m} E_i = I$ (the $n \times n$ identity matrix), where $\mathcal{N}_i, i = 1, \cdots, m$ are nonempty subsets of $\mathcal{N}, \mathcal{N} = \{1, \cdots, n\}$ satisfying $\mathcal{N} = \bigcup_{i=1}^{m} \mathcal{N}_i$, see also [4]. Then the (parallel linear) multisplitting iterative method for solving the linear system of equations

$$Ax = b \tag{1.1}$$

is

$$M_i x_i^{(k)} = N_i x^{(k-1)} + b, \quad i = 1, \cdots, m; \quad k = 1, \cdots, \tag{1.2}$$

$$x^{(k)} = \sum_{i=1}^{m} E_i x_i^{(k)}. \tag{1.3}$$

In particular, the above method is called a nonoverlapping multisplitting iterative method if $\mathcal{N}_{i_1} \cap \mathcal{N}_{i_2} = \emptyset$ $(i_1 \neq i_2)$.

Subsequently, many authors studied the above technique for the cases where A is an M-matrix, an H-matrix or a symmetric positive definite matrix respectively, we refer to [1,4,6,9, 11,13,17] and the references therein. The idea of minimizing the norm of either the error or the residual so that the numerically optimal value of the iteration parameter is determined, first introduced in [2], used to compute a numerically optimal relaxation parameter for the successive overrelaxation (SOR) iteration methods for solving the system of linear equations. Based on the standard quadratic programming technique, the authors of this paper and their collaborators [14,15] seem to be the first to introduce the auto-optimal weighting matrices $E_i^{(k)}, i = 1, \cdots, m;\ k = 1, \cdots$ for parallel multisplitting iterative methods. The self-adaptive weighting matrices enable more approximate to the exact solution for k-step iteration. These methods, however, just as introduced in [14] and [15], only one processor of the multiprocessor system to compute the global optimization model at every iteration step, the other $m - 1$ processors must be in the waiting state until one of all these processors has finished its optimization task.

As is well known, one of the best accelerated methods is the Chebyshev semi-iteration, in which the optimum parameter ω is obtained by the Chebyshev polynomial. The one most recent result may be found the quasi-Chebyshev accelerated (QCA) method to convergent splitting iteration proposed in [16]. The method is, in spirit, analogous to the Chebyshev semi-iteration but the optimum parameter ω is generated by optimization model for solving the linear systems. These motivated us to accelerate the parallel multisplitting iterative method, resulting in a parallel quasi-Chebyshev accelerated (abbreviated as PQCA) method to the nonoverlapping multisplitting iterative method for the linear systems when the coefficient matrices are H-matrices or symmetric positive definite matrices. To make full use of the efficiency of a multiprocessor system and overcome the drawbacks of those methods in [14,15], we further divide the global optimization model into m sub-models and hence, the parallel computing is achieved in this paper.

The PQCA method determines the optimum parameters $\alpha^{(i)}, i = 1, \cdots, m$ through minimizing either the l_2-norm of the residual when the coefficient matrix of (1.1) is a symmetric positive definite matrix, or the l_1-norm of the residual when the coefficient matrix of (1.1) is an H-matrix, at each step of their iterates, at each processor of multiprocessor system, with a reasonably extra cost. In actual computations, that shows better numerical behaviors than the Method in [9] for both the symmetric positive definite matrix and the H-matrix. Numerical experiments show that the new PQCA method is feasible, efficient and robust for solving large sparse system of linear equations (1.1).