Journal of Computational Mathematics
Vol.32, No.4, 2014, 412–441.

http://www.global-sci.org/jcm
doi:10.4208/jcm.1401-m4214

STATIC REGIME IMAGING OF LOCATIONS OF CERTAIN 3D ELECTROMAGNETIC IMPERFECTIONS FROM A BOUNDARY PERTURBATION FORMULA*

Séraphin M. Mefire
IECL, CNRS UMR 7502, Université de Lorraine, Faculté des Sciences et Technologies de Nancy, 54506 Vandœuvre-lès-Nancy Cedex, France
Email: seraphin.mefire@univ-lorraine.fr

Abstract

We are concerned, in a static regime, with an imaging approach of the locations in a three-dimensional bounded domain of certain electromagnetic imperfections. This approach is related to Electrical Impedance Tomography and makes use of a new perturbation formula in the electric fields. We present two localization procedures, from a Current Projection method that deals with the single imperfection context and an inverse Fourier process that is devoted to multiple imperfections configurations. These procedures extend those that were described in our previous work, since operating for a broader class of settings. Namely, the localization is additionally performed for certain purely electric imperfections, as established from numerical simulations.

Mathematics subject classification: 65N21, 65N30, 78A25.
Key words: Inverse problems, Maxwell equations, Electric fields, Inhomogeneities, Electrical Impedance Tomography, Current Projection method, FFT, Numerical boundary measurements, Random noise, Edge elements, Least square systems, Incomplete Modified Gram-Schmidt preconditioning.

1. Introduction

In the context of certain applications, the imaging process mainly consists of determining from boundary measurements the locations of a finite number of defects contained in a bounded domain, and a full reconstruction of each of these defects is not necessary. Diverse reasons can motivate this determination of locations only: a priori knowledge of shapes of the defects, effective reconstruction time cost, smallness of the defects, ... Several recent works have been devoted to such a determination when the common order of magnitude of the diameters of these defects is small. The approach in these works (see e.g. [1-5,12,18]), related to Electrical Impedance Tomography, consists of localizing the defects from a particular combination of an asymptotic formula for perturbations in the potential or field, in the presence of defects, with an inversion process. Such a formula makes use of polarization tensors [3], associated with defects, and the consideration of direct inversion processes is well suited. This approach has been developed in various situations: small conductivity inhomogeneities, small elastic inhomogeneities, ...

In the presence of three-dimensional settings of small electromagnetic inhomogeneities and in the frequency regime, H. Ammari, M. Vogelius and D. Volkov [6] have proposed the asymptotic formula for perturbations in the electromagnetic fields. This tool has been thus considered by

* Received July 10, 2012 / Revised version received July 19, 2013 / Accepted January 15, 2014 / Published online July 3, 2014 /

M. Asch and S.M. Mefire [8, 9] for performing the numerical localization in a three-dimensional bounded domain of these inhomogeneities, in diverse experimental configurations. Of course, in the static regime, and in contrast to the frequency regime, the localization is no longer subject to a suitable choice of frequency or of a range of frequencies. However, the inhomogeneities that are characterized by a complex electric permittivity are not considered in the localization from the static regime. In [14, 15], we deal with the static regime in several experimental configurations. The two localization procedures introduced in these works are based on a limit model in electric field and a limit perturbation model, in combination with the Current Projection method (for the single inhomogeneity context) or with an inverse Fourier method (for the multiple inhomogeneities context). These limit models are respectively obtained by letting the frequency vanish in the time-harmonic Maxwell equations and in the asymptotic formula for perturbations in the electromagnetic fields proposed in [6]. It results from [14,15] that these procedures achieve uniquely the numerical localization of inhomogeneities involving a magnetic contrast; purely electric inhomogeneities, namely inhomogeneities of real electric permittivity and without magnetic characteristics, cannot thus be localized from these procedures. This observation leads us to an essential question, that of knowing whether, from this limit model in electric field, both purely magnetic and purely electric inhomogeneities can be localized? The numerical treatment of this question constitutes here the main objective.

Unlike in [14,15], we will deal here with an asymptotic formula for perturbations that derives directly from the limit model in electric field.

This work is subdivided into six sections. In Section 2, after recalling the limit model in electric field following [14], we introduce an asymptotic formula that allows to study perturbations in the electric fields due to the presence of small electromagnetic inhomogeneities in a three-dimensional bounded domain Ω. This asymptotic formula contains information on the electric contrast as well as on the magnetic contrast relatively to Ω, and constitutes a basis for some approximate inversion techniques. Since boundary measurements, initiated by electric currents applied on the boundary of Ω, shall be generated in numerical simulations and from this asymptotic formula, we briefly recall in Section 3, following [14], the (direct) computation of the electric field then required in the evaluation of each measurement. This computation is achieved by a least squares approach, with the help of Nédélec's edge elements and nodal finite elements. In Section 4, we present two localization procedures resulting from the combination of the asymptotic formula with each of the following inversion processes: the Current Projection method and an inverse Fourier method. Each procedure uses boundary measurements (in a finite number) as data and is employed in the single inhomogeneity context. Only the procedure considering an inverse Fourier method is required for multiple inhomogeneities configurations. As a result, these localization procedures extend those that were introduced in [14], since dealing with the case of purely magnetic inhomogeneities in an identical way and furthermore numerically suitable for the context of purely electric inhomogeneities. We describe in Section 5 localization results obtained from various settings and compare some results in the single inhomogeneity context. Finally, some conclusions are reported in Section 6.

2. Perturbation Formula in the Electric Fields

We start by considering some notation, before introducing the perturbation formula in the electric fields.