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RESTRICTED ADDITIVE SCHWARZ METHOD FOR A KIND OF NONLINEAR COMPLEMENTARITY PROBLEM *

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Abstract

In this paper, a new Schwarz method called restricted additive Schwarz method (RAS) is presented and analyzed for a kind of nonlinear complementarity problem (NCP). The method is proved to be convergent by using weighted maximum norm. Besides, the effect of overlap on RAS is also considered. Some preliminary numerical results are reported to compare the performance of RAS and other known methods for NCP.

Mathematics subject classification: 65N30, 65M60
Key words: Nonlinear complementarity problem, Nonlinear source term, Restricted additive Schwarz method, Weighted max norm.

1. Introduction

We consider the following finite-dimensional nonlinear complementarity problem (NCP): find $u \in R^n$ such that

$$u \geq \phi, \quad F(u) \geq 0, \quad (u-\phi)^T F(u) = 0, \tag{1.1}$$

where $\phi \in R^n$, $F = Au + f(u)$ with an $M-$matrix A, and $\partial f/\partial u \geq 0$. This kind of problem can be arised from free boundary problem with nonlinear source terms, e.g. the diffusion problem involving Michaelis-Menten or second-order irreversible reactions, see, e.g., [8,15].

It is well known that domain decomposition method is a kind of very important method for PDEs since 1980's. It has many advantages, for example it is easy to be parallelized on parallel machines, and it is effective for large scale problem. Moreover, the convergence rate will not be deteriorated with the refinement of the mesh when it is applied to discretized differential equations. Researches in this field for the solution of variational inequalities and complementarity problems were also fruitful in the last decade. We refer the reader to [1,2,9-11,13,16,17,20-22] and the extensive references therein. In [6], a new variant Schwarz method called restricted additive Schwarz method (RAS) was proposed for general sparse linear systems. This method attracts much attention, since it reduces communication time while maintaining the most desirable used in practice. Up to now, much effort have been made to study the convergence theory for RAS, and extended RAS for some other kinds of linear systems, see, e.g., [4,5,7,9] and the references therein. [18] presented the RAS method for a kind of NCP, but

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the authors did not prove the convergence of the proposed method. The purpose of this paper is to extend the RAS method to NCP (1.1) and establish its convergence results. Moreover, we discuss the effect of overlap on proposed method.

The paper is organized as follows: in Section 2, we give some preliminaries and present the RAS method. In Section 3, we estimate the weighted max-norm bounds for iteration errors and establish global convergence theorem for RAS. In Section 4, we discuss the effect of overlap on the RAS method. In Section 5, we present some numerical results and give some conclusions in Section 6.

2. Restricted Additive Schwarz Method

In this section, we present the RAS method for solving problem (1.1). As in [6] and [9], we consider m nonoverlapping subspaces $V_{i,0}$, $i = 1, \cdots, m$, which are spanned by columns of the identity I over R^n. Let $S = \{1, 2, \cdots, n\}$ and let

$$S = \cup_{i=1}^{m} S_{i,0}$$

be a partition of S into m disjoint, nonempty subsets. Let $\{S_{i,1}\}$ be the one-overlap partition of S which is obtained by adding those indices to $S_{i,0}$ which correspond to nodes lying at distance 1 or less from those nodes corresponding to $S_{i,0}$. Using the idea recursively, we can define $\delta-$overlap partition of S, $S = \cup_{i=1}^{m} S_{i,\delta}$, where $S_{i,0} \subset S_{i,\delta}$, with δ level of overlaps with its neighboring subsets. Hence, we have a nested sequence of larger sets $S_{i,\delta}$ with

$$S_{i,0} \subseteq S_{i,1} \subseteq S_{i,2} \subseteq \cdots \subseteq S = \{1, 2, \cdots, n\}. \tag{2.1}$$

For $\delta > 0$, the sets $S_{i,\delta}$ are not necessarily pairwise disjoint, i.e., we have introduced overlap. This approach is adequate in discretizations of PDEs where the indices correspond to the nodes of the discretization mesh, and also is valid for problem (1.1).

Let $n_{i,\delta} = |S_{i,\delta}|$ denote the cardinality of the set $S_{i,\delta}$. For each nested sequence from (2.1), we can find a permutation matrix π_i on $\{1, 2, \cdots, n\}$ with the property that for all $\delta \geq 0$ we have $\pi_i(S_{i,\delta}) = \{1, 2, \cdots, n_{i,\delta}\}$. Let $R_{i,\delta} : R^n \to R^{n_{i,\delta}}$ be the restriction operator. $R_{i,\delta}$ is an $n_{i,\delta} \times n$ matrix with rank $(R_{i,\delta})= n_{i,\delta}$ whose rows are precisely those row j of the identity for which $j \in S_{i,\delta}$. Its transpose $R_{i,\delta}^T : R^{n_{i,\delta}} \to R^n$ is a prolongation operator. Formally, such a matrix $R_{i,\delta}$ can be expressed as

$$R_{i,\delta} = [I_{i,\delta} \quad O]\pi_i \tag{2.2}$$

with $I_{i,\delta}$ the identity on $R^{n_{i,\delta}}$. We define the weighting matrices

$$E_{i,\delta} = R_{i,\delta}^T R_{i,\delta} = \pi_i^T \begin{bmatrix} I_{i,\delta} & O \\ O & O \end{bmatrix} \pi_i \in R^{n\times n} \tag{2.3}$$

and the subspaces

$$V_{i,\delta} = \mathrm{range}(E_{i,\delta}), \quad i = 1, 2, \cdots, m.$$

Note the inclusion $V_{i,\delta} \subseteq V_{i,\delta'}$ for all $\delta \leq \delta'$, and in particular, $V_{i,0} \subseteq V_{i,\delta}$ for all $\delta \geq 0$. Furthermore, the images of the bases of $V_{i,\delta}$ under the prolongation operator $R_{i,\delta}^T$ are linearly independent unit elements in R^n, and we can verify the images of $R_{i,\delta}^T$ with the subspaces $V_{i,\delta}$. For matrix A, let $A_{i,\delta} = R_{i,\delta} A R_{i,\delta}^T$ denote the restriction of A to $V_{i,\delta}$. By (2.2), we have matrix