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TWO-STEP MODULUS-BASED SYNCHRONOUS MULTISPLITTING ITERATION METHODS FOR LINEAR COMPLEMENTARITY PROBLEMS*

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Abstract

To reduce the communication among processors and improve the computing time for solving linear complementarity problems, we present a two-step modulus-based synchronous multisplitting iteration method and the corresponding symmetric modulus-based multisplitting relaxation methods. The convergence theorems are established when the system matrix is an H_+-matrix, which improve the existing convergence theory. Numerical results show that the symmetric modulus-based multisplitting relaxation methods are effective in actual implementation.

Mathematics subject classification: 65F10, 68W10, 90C33.
Key words: Linear complementarity problem, Modulus-based method, Matrix multisplitting, Convergence.

1. Introduction

Given a real matrix $A \in \mathbb{R}^{n\times n}$ and a real vector $q \in \mathbb{R}^n$, the linear complementarity problem abbreviated as $\mathrm{LCP}(q, A)$ is to find a pair of real vectors $r, z \in \mathbb{R}^n$ such that

$$r := Az + q \geq 0, \quad z \geq 0 \quad \text{and} \quad z^T(Az + q) = 0.$$

The linear complementarity problem has extensive applications in the field of economy and engineering; see [11,14]. The modulus method is one of the classic iteration methods for solving linear complementarity problems; see, e.g., [13,21,24]. More recently, Hadjidimos and Tzoumas presented the extrapolated modulus algorithms in [17,18], and Bai presented the modulus-based matrix splitting iteration method in [3]. These two new methods are very effective and practical in numerical computation.

For large sparse linear complementarity problems arising in the engineering applications, the multisplitting iterative methods are powerful tools to enlarge the scale of problem and speed up the computation; see, e.g., [1, 2, 4, 5, 7, 12, 22]. Recently, by an equivalent reformulation of the linear complementarity problem into a system of fixed-point equations, Bai and Zhang have constructed the *modulus-based synchronous multisplitting* (**MSM**) iteration methods in [7], which are suitable to be implemented parallelly on multiprocessor systems. As the communication among processors is much more time-consuming than the computation, we intend to reduce

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the communication by making full use of the previous iteration and communication. To this end, we present the two-step modulus-based synchronous multisplitting iteration methods as well as their relaxed variants in this paper, which consist of two sweeps at each iteration step. We remark that these two-step methods are different from the *two-stage* methods presented in [8, 27], which are inner/outer iteration methods aimed to solve the outer iteration efficiently.

The remaining part of this paper is organized as follows: In Section 2, we introduce some notations and briefly review the MSM iteration methods. In Section 3, we propose the two-step modulus-based synchronous multisplitting iteration methods as well as their relaxed variants. In Section 4, we prove their convergence when the system matrix is an H_+-matrix. Numerical results are given in Section 5. Finally, we make a conclusion in Section 6.

2. Notations and Preliminaries

For $A = (a_{ij}) \in \mathbb{R}^{m\times n}$ and $B = (b_{ij}) \in \mathbb{R}^{m\times n}$, we write $A \geq B$ ($A > B$) if $a_{ij} \geq b_{ij}$ ($a_{ij} > b_{ij}$) hold for all $1 \leq i \leq m$ and $1 \leq j \leq n$. If O is the null matrix and $A \geq O$ ($A > O$), we say that A is a nonnegative (positive) matrix. $|A|$ and A^T denote the absolute value and the transpose of the matrix A, respectively.

For a square matrix $A = (a_{ij}) \in \mathbb{R}^{n\times n}$, we denote its spectral radius and diagonal part by $\rho(A)$ and $\mathrm{diag}(A)$, respectively. Its comparison matrix $\langle A\rangle = (\langle a_{ij}\rangle)$ is defined by $\langle a_{ij}\rangle = |a_{ij}|$ if $i = j$ and $\langle a_{ij}\rangle = -|a_{ij}|$ if $i \neq j$. It is called an M-matrix if its off-diagonal entries are all non-positive and $A^{-1} \geq O$, an H-matrix if its comparison matrix $\langle A\rangle$ is an M-matrix, and an H_+-matrix if it is an H-matrix with positive diagonal entries [2, 9, 25]. Note that if A is an H_+-matrix, then $\rho(D^{-1}|B|) < 1$, where $D = \mathrm{diag}(A)$ and $B = D - A$; see [9]. In this paper, we focus on the case that A is an H_+-matrix, which is a sufficient condition for LCP(q, A) to possess a unique solution for any q.

If A is an M-matrix and Λ is a positive diagonal matrix, then $A \leq B \leq \Lambda$ implies that B is an M-matrix. If A is an H-matrix, then A is nonsingular and $|A^{-1}| \leq \langle A\rangle^{-1}$; see, e.g., [9, 15]. The splitting $A = M - N$ is called an H-compatible splitting if it satisfies $\langle A\rangle = \langle M\rangle - |N|$; see, e.g., [16].

Lemma 2.1. ([19,20]). *Let $M = (m_{ij}) \in \mathbb{R}^{n\times n}$ be a strictly diagonally dominant matrix. Then*

$$\left\|M^{-1}N\right\|_\infty \leq \max_{1\leq i\leq n} \frac{\sum\limits_{j=1}^{n} |n_{ij}|}{|m_{ii}| - \sum\limits_{j\neq i} |m_{ij}|}$$

holds for any matrix $N = (n_{ij}) \in \mathbb{R}^{n\times n}$.

Lemma 2.2. ([3]). *Let $A = M - N$ be a splitting of the matrix $A \in \mathbb{R}^{n\times n}$, Ω be a positive diagonal matrix, and γ be a positive constant. For the LCP (q, A), the following statements hold true:*

(i) if (z, r) is a solution of the LCP(q, A), then $x = \frac{1}{2}\gamma(z - \Omega^{-1}r)$, with $|x| = \frac{1}{2}\gamma(z + \Omega^{-1}r)$, satisfies the implicit fixed-point equation

$$(\Omega + M)x = Nx + (\Omega - A)|x| - \gamma q; \tag{2.1}$$