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ON THE PROBLEM OF INSTABILITY IN THE DIMENSIONS OF SPLINE SPACES OVER T-MESHES WITH T-CYCLES*

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Abstract

The T-meshes are local modification of rectangular meshes which allow T-junctions. The splines over T-meshes are involved in many fields, such as finite element methods, CAGD etc. The dimension of a spline space is a basic problem for the theories and applications of splines. However, the problem of determining the dimension of a spline space is difficult since it heavily depends on the geometric properties of the partition. In many cases, the dimension is unstable. In this paper, we study the instability in the dimensions of spline spaces over T-meshes by using the smoothing cofactor-conformality method. The modified dimension formulas of spline spaces over T-meshes with T-cycles are also presented. Moreover, some examples are given to illustrate the instability in the dimensions of the spline spaces over some special meshes.

Mathematics subject classification: 65D07.
Key words: Spline space, Smoothing cofactor-conformality method, Instability in the dimension, T-meshes.

1. Introduction

As we known, the Non-Uniform Rational B-Spline (NURBS) is commonly used in computer-aided design (CAD), manufacturing (CAM), and engineering (CAE). However, NURBS has a weakness that the control points must lie topologically on a rectangular grid, since NURBS is based on a tensor product structure. Moreover, NURBS models contain a large number of superfluous control points, which are big burdens to modeling systems [1, 2].

To overcome the limitation of NURBS, Sederberg et al. [1] invented T-spline, which is a point-based spline defined on T-mesh. This type of spline supports many valuable operations within a consistent framework. The local refinement of T-spline is dependent on the structure of the mesh, the refinement can be performed by a recursive procession as the algorithm given in [2]. Recently, a class of T-splines, called Analysis-suitable T-splines, has been proposed, which is linearly independent and forms a partition of unity [3, 4]. Scoot et al. [4] also gave a greedy algorithm to achieve the local refinement for Analysis-suitable T-splines.

The dimension is one of the crucial characteristics of a spline space that must be analyzed. In general, the degree of a bivariate spline space includes two cases, one is bi-degree, the other is total degree. The spline space over T-mesh $S(m, n, \alpha, \beta, \mathcal{T})$ was introduced in [5], which is a bi-degree (m, n) piecewise polynomial spline space over T-mesh $\mathcal{T}$ with smoothness order α and β in two directions. Deng et al. [5] firstly gave the dimension of spline space $S(m, n, \alpha, \beta, \mathcal{T})$ by using the B-net method when the T-mesh has no cycles with $m \geq 2\alpha + 1$ and $n \geq 2\beta + 1$.

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Huang et al. [6] derived an equivalent dimension formula in a different form with the smoothing cofactor-conformality method [7, 8]. Li et al. [9] improved the dimension formula of the same spline space by using the smoothing cofactor-conformality method with a constraint depending on the order of the smoothness, the degree of the spline functions and the structure of the T-meshes as well. For the second case of total degree, Li et al. [10] discussed the dimension of spline space $S^{\mu}_{k}(\Delta_{QR})$, which is a spline space with total degree k and smoothness μ over quasi-rectangular meshes Δ_{QR}. Further, Li et al. [11] discussed the stability of the dimensions of general spline spaces based on the analysis on the conformality condition at one interior vertex. However, in these papers, a special structure of the mesh, the cases with T-cycles were discussed insufficiently.

Li et al. [12] discovered that the dimension of the associated spline space is unstable over some particular T-meshes, i.e, the dimension not only depends on the topological information of the T-mesh but also depends on the geometry construction of the T-mesh. The special T-meshes presented in [12] do not satisfy the conditions of the dimension formula given in [9]. Inspired by [12], we reconsider the dimension of spline space over T-meshes with T-cycles. We find that the dimensions of $S(m, n, \alpha, \beta, \mathcal{T})$ may be unstable when the T-meshes contain T-cycles though the conditions of the dimension formula given in [9] are satisfied. Moreover, we point out that the dimensions of $S^{\mu}_{k}(\mathcal{T})$ are also unstable when the T-meshes contain T-cycles though the conditions in [10] are satisfied. Then we give the corrected dimension formulas over T-meshes with T-cycles.

The paper is organized as follows. We give some basic definitions and theorems about splines in Section 2. In Section 3, we discuss the instability in the dimensions of spline spaces over T-meshes and modify the dimension formulas in [9] and [10]. Some examples are presented in Section 4. Finally, conclusions are drawn in Section 5.

2. Preliminaries

In this section, we will give some definitions and notations.

2.1. T-mesh

We use the same definitions and notations of T-mesh as in [10].

Given a rectangular mesh Δ_R (see Fig. 2.1(a)), we modify it locally to get a new mesh (as shown in Fig. 2.1(b)). There are three kinds of interior mesh segments, here interior mesh segment means the longest possible interior line segment.

a) cross-cut: both of its endpoints lie on the boundary of the mesh, e.g. v_1v_2 in Fig. 2.1(b).

b) ray: only one of its endpoints lies on the boundary of the mesh, e.g. v_3v_{11} in Fig. 2.1(b).

c) Truncated-segment (or T-segment): both of its endpoints are interior vertices, e.g. v_4v_{18} in Fig. 2.1(b).

We define three kinds of interior vertices.

1) free-vertex: the intersection point of cross-cuts or rays, e.g. v_{21}, v_{22}, v_{27} in Fig. 2.1(b).

2) mono-vertex: the intersection point of one T-segment and one cross-cut or one ray, e.g. v_5, v_9, v_{16} in Fig. 2.1(b).