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MULTI-SYMPLECTIC FOURIER PSEUDOSPECTRAL METHOD FOR A HIGHER ORDER WAVE EQUATION OF KDV TYPE*

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Abstract

The higher order wave equation of KdV type, which describes many important physical phenomena, has been investigated widely in last several decades. In this work, multi-symplectic formulations for the higher order wave equation of KdV type are presented, and the local conservation laws are shown to correspond to certain well-known Hamiltonian functionals. The multi-symplectic discretization of each formulation is calculated by the multi-symplectic Fourier pseudospectral scheme. Numerical experiments are carried out, which verify the efficiency of the Fourier pseudospectral method.

Mathematics subject classification: 65N30.
Key words: The higher order wave equation of KdV type, Multi-symplectic theory, Fourier pseudospectral method, Local conservation laws.

1. Introduction

As is well known, the Korteweg-de Vries (KdV) equation represents a first order approximation in the study of long wavelength, small amplitude waves of inviscid and incompressible fluid. In this paper, we consider the higher order wave equation of KdV type [1-5]

$$v_t - \frac{3}{2}\beta\rho_2 v_{xxt} + \beta(1 - \frac{3}{2}\rho_2)v_{xxx} + \alpha v v_x - \frac{1}{2}\alpha\beta\rho_2(v v_{xxx} + 2v_x v_{xx}) = 0. \qquad (1.1)$$

Exact solutions for some special set of parameters of the equation (1.1) have been studied by many authors. In [6], Long et al. proved the existence of all traveling wave solutions, and obtained two explicit parametric representations of periodic solutions of equation (1.1). In [7], Long et al. obtained the explicit and implicit traveling wave solutions of equation (1.1). In [8], Rui et al. obtained new traveling wave solutions, explicit solutions of parametric type of equation (1.1) by integral bifurcation method.

Several numerical methods for Eq. (1.1) have been studied in recent years. In [2], the equation (1.1) has been solved numerically by pseudospectral method. However, since the numerical method used in [2] is not structure-preserving, all the qualitative behaviors such as norm conservation has been lost in the discretization. In 1984, Feng [9] proposed a structure-preserving algorithm to compute partial differential equations from the view point of symplectic geometry. However, the disadvantage of this method is that it is global. To overcome this limitation, Bridge and Reich [12] presented a multi-symplectic algorithm based on a multi-symplectic structure of some partial differential equations.

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One of the most popular multi-symplectic algorithms is Fourier pseudospectral method which has been proven very powerful for periodic initial value problems with constant coefficients. The Fourier pseudospectral method is successfully applied to model wave propagation. Note that Eq. (1.1) represents a multi-symplectic Hamiltonian system. This paper is organized as follows. In Section 2, the multi-symplectic Hamiltonian formulations for (1.1) are established and some conservation properties are obtained. In Section 3, devotes to the construction of multi-symplectic Fourier pseudospectral method. In Section 4, numerical experiments are given.

2. Multi-symplectic Structure for (1.1)

By the multi-symplectic theory [12-31], we know that many conservative systems can be written as multi-symplectic Hamiltonian equations

$$Mz_t + Kz_x = \nabla_z S(z), \tag{2.1}$$

where $M,\ K \in R^{d\times d}$ are the skew-symmetric matrices, $z(x,t)$ is the vector of state variables, $S : R^n \to R$ is a smooth function, and $\nabla_z S(z)$ denotes the gradient of the function $S = S(z)$ with respect to variable z.

Eq. (2.1) satisfied the multi-symplectic conservation law (MSCL)

$$\frac{\partial}{\partial t}w + \frac{\partial}{\partial x}k = 0, \tag{2.2}$$

where

$$w = \frac{1}{2}dz \wedge Mdz, \quad k = \frac{1}{2}dz \wedge Kdz. \tag{2.3}$$

Moreover, Eq. (2.1) satisfies the local energy conservation law (LECL)

$$\frac{\partial}{\partial t}E + \frac{\partial}{\partial x}F = 0, \tag{2.4}$$

and local momentum conservation law (LMCL)

$$\frac{\partial}{\partial t}I + \frac{\partial}{\partial x}G = 0, \tag{2.5}$$

where

$$E = S(z) - \frac{1}{2}z^T Kz_x, \quad F = \frac{1}{2}z^T Kz_t, \quad I = \frac{1}{2}z^T Mz_x, \quad G = S(z) - \frac{1}{2}z^T Mz_t.$$

By introducing new variables $v = u_x$, the higher order wave equation of KdV type (1.1) can be written as

$$u_{xt} - \frac{3}{2}\beta\rho_2 u_{xxxt} + \beta(1 - \frac{3}{2}\rho_2)u_{xxxx} + \alpha u_x u_{xx} - \frac{1}{2}\alpha\beta\rho_2(u_x u_{xxxx} + 2u_{xx}u_{xxx}) = 0. \tag{2.6}$$

If we define the Lagrange function

$$\begin{aligned} &L(x,t,u,u_x,u_t,u_{xx},u_{xt},u_{tt}) \\ &= -\frac{1}{2}u_x u_t - \frac{3}{4}\beta\rho_2 u_{xx}u_{xt} + \frac{\beta}{2}\left(1 - \frac{3}{2}\rho_2\right)u_{xx}^2 - \frac{1}{6}\alpha v_x^3 - \frac{1}{4}\alpha\beta\rho u_x u_{xx}^2, \end{aligned} \tag{2.7}$$