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NONLINEAR LAGRANGIANS FOR NONLINEAR PROGRAMMING BASED ON MODIFIED FISCHER-BURMEISTER NCP FUNCTIONS*

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Abstract

This paper proposes nonlinear Lagrangians based on modified Fischer-Burmeister NCP functions for solving nonlinear programming problems with inequality constraints. The convergence theorem shows that the sequence of points generated by this nonlinear Lagrange algorithm is locally convergent when the penalty parameter is less than a threshold under a set of suitable conditions on problem functions, and the error bound of solution, depending on the penalty parameter, is also established. It is shown that the condition number of the nonlinear Lagrangian Hessian at the optimal solution is proportional to the controlling penalty parameter. Moreover, the paper develops the dual algorithm associated with the proposed nonlinear Lagrangians. Numerical results reported suggest that the dual algorithm based on proposed nonlinear Lagrangians is effective for solving some nonlinear optimization problems.

Mathematics subject classification: 90C30, 49M37, 65K05.
Key words: nonlinear Lagrangian, nonlinear Programming, modified Fischer-Burmeister NCP function, dual algorithm, condition number.

1. Introduction

Consider the nonlinear programming problem with inequality constraints of the form

$$\begin{array}{ll} \text{minimize} & f_0(x) \\ \text{subject to} & f_i(x) \geq 0,\ i = 1, \cdots, m, \end{array} \tag{NLP}$$

where $x \in \mathbf{R}^n, f_i(x) : \mathbf{R}^n \to \mathbf{R}^1, i = 0, \cdots, m$ are real-valued functions.

The classical (linear) Lagrangian of (NLP) is defined by

$$L(x, u) = f_0(x) - \sum_{i=1}^{m} u_i f_i(x), \tag{1.1}$$

which plays important roles in describing the optimality conditions for (NLP) and designing algorithms for finding solutions to (NLP). For convex programming, the saddle point theory

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can be established in terms of the classical Lagrangian and dual algorithms based on solving $\min L(x, u^k)$ for some u^k can be developed as well. For nonconvex nonlinear programming, $L(x, u^k)$ is usually not convex even for u^k close to u^* and x in a neighborhood of x^*, where (x^*, u^*) is a Kuhn-Tucker point for (NLP), and this encounters difficulties in numerical implementations. To solve this problem, Hestenes [9], Powell [19] introduced the augmented Lagrangian for problems with equality constraints and Rockafellar [21-23] developed the augmented Lagrangian for problems with both equality and inequality constraints. For more details on the augmented Lagrange method we refer to Bertsekas [3] or Bertsekas [4].

As nonlinear Lagrangians can be used to develop dual algorithms for nonlinear programming, requiring no restrictions on the feasibility of primal variables, important contributions on this topic have been done by many authors in recent years.

For convex programming, Polyak and Teboulle [18] discussed a class of Lagrange functions of the form

$$G(x, u, \mu) = f_0(x) - \frac{1}{\mu} \sum_{i=1}^{m} u_i \psi(\mu f_i(x)), \tag{1.2}$$

where $\mu > 0$ is penalty parameter and ψ is twice continuous differentiable function. Based on Log-Sigmoid function, Polyak [16] developed a specific nonlinear rescaling (NR) method and estimated its convergence rate. Furthermore, Polyak and Griva [17] proposed a general primal-dual nonlinear rescaling (PDNR) method for convex optimization with inequality constraints, and Griva and Polyak [8] developed a general Primal-dual nonlinear rescaling method with dynamic scaling parameter update. Besides the works by Polyak and his coauthors, Auslender et al. [1] and Ben-Tal and Zibulevsky [2] studied other nonlinear Lagrangians and obtained interesting convergence results for convex programming problems, too.

For nonconvex programming, a class of nonlinear Lagrangians for inequality constrained problems, leading to unconstrained saddle point problems, was introduced by Mangasarian [13]; Charalambous [5] gave the minimum p-th function; Bertsekas [3] proposed the exponential Lagrangian as follows:

$$F(x, u, k) = f_0(x) - k^{-1} \sum_{i=1}^{m} u_i [1 - e^{-k f_i(x)}]; \tag{1.3}$$

Polyak [14] gave two modified barrier functions, namely, modified Frish's function

$$F(x, u, k) = \begin{cases} f_0(x) - k^{-1} \sum_{i=1}^{m} u_i \ln(k f_i(x) + 1), & x \in \mathrm{int}\Omega_k, \\ +\infty, & x \notin \mathrm{int}\Omega_k, \end{cases} \tag{1.4}$$

and modified Carroll's function

$$C(x, u, k) = \begin{cases} f_0(x) - k^{-1} \sum_{i=1}^{m} u_i [1 - (k f_i(x) + 1)^{-1}], & x \in \mathrm{int}\Omega_k, \\ +\infty, & x \notin \mathrm{int}\Omega_k, \end{cases} \tag{1.5}$$

where $k > 0$ is parameter and $\Omega_k = \{x | 1 + k f_i(x) \geq 0, i = 1, \cdots, m\}$; Li [11] constructed the p-th power Lagrangian

$$L_p(x, u) = [f_0(x)]^p - \sum_{i=1}^{m} u_i \{[\bar{f}_i(x)]^p - b_i^p\}, \tag{1.6}$$

and partial p-th power Lagrangian

$$L_p(x, u) = f_0(x) - \sum_{i=1}^{m} u_i \{[\bar{f}_i(x)]^p - b_i^p\}, \tag{1.7}$$