Journal of Computational Mathematics
Vol.33, No.4, 2015, 428–442.
http://www.global-sci.org/jcm
doi:10.4208/jcm.1504-m4493

SOME PROPERTIES FOR ANALYSIS-SUITABLE T-SPLINES*

Xin Li
School of Mathematical Science, University of Science and Technology of China, Hefei, Anhui, China
Email: lixustc@ustc.edu.cn

Abstract

Analysis-suitable T-splines (AS T-splines) are a mildly topological restricted subset of T-splines which are linear independent regardless of knot values [1–3]. The present paper provides some more iso-geometric analysis (IGA) oriented properties for AS T-splines and generalizes them to arbitrary topology AS T-splines. First, we prove that the blending functions for analysis-suitable T-splines are locally linear independent, which is the key property for localized multi-resolution and linear independence for non-tensor-product domain. And then, we prove that the number of T-spline control points contribute each Bézier element is optimal, which is very important to obtain a bound for the number of non zero entries in the mass and stiffness matrices for IGA with T-splines. Moreover, it is found that the elegant labeling tool for B-splines, blossom, can also be applied for analysis-suitable T-splines.

Mathematics subject classification: 65D07.
Key words: T-splines, Linear independence, iso-geometric analysis, Analysis-suitable T-splines.

1. Introduction

T-splines [4, 5] have been used to solve many limitations inherent in the industry standard NURBS representation, such as watertightness [4, 6], trimmed NURBS conversion [7] and local refinement [5, 8]. Thus, T-splines have proved to be an important technology across several disciplines including industrial, architectural and engineering design, manufacturing and engineering analysis. Especially, these capabilities make T-splines attractive for both geometric modeling and iso-geometric analysis (IGA), which use the smooth spline basis that defines the geometry as the basis for analysis. IGA was introduced in [9] and described in detail in [10]. Traditional design-through-analysis procedures such as geometry clean-up, defeaturing, and mesh generation are simplified or eliminated entirely. Additionally, the higher-order smoothness provides substantial gains to analysis in terms of accuracy and robustness of finite element solutions [11–13]. The use of T-splines as a basis for IGA has gained widespread attention [8, 14–16].

However, [17] discovered an example of a T-spline with *linear dependent* blending functions, which means that not all T-splines are suitable as a basis for IGA. Thus, an important development in the evolution of IGA was the advent of analysis-suitable T-splines (AS T-splines), a mildly topological restricted subset of T-splines. AS T-splines are optimized to meet the needs both for design and analysis [1, 8]. Such T-splines inherit all the good properties from T-splines, such as watertightness, NURBS compatible, convex hull, and affine invariant. Unlike the general T-splines, such T-splines are guaranteed to be linearly independent [1], the

* Received February 18, 2014 / Revised version received September 9, 2014 / Accepted April 8, 2015 / Published online July 17, 2015 /

polynomial blending functions for such T-splines sum identically to one for an admissible T-mesh [18,19] and the T-spline space can be characterized in terms of piecewise polynomials [18]. Furthermore, algorithms have been developed whereby local refinement of such T-splines is well contained [8].

The present paper studies several more IGA-oriented properties for analysis-suitable T-splines, including local linear independence, the number of control points which contribute one Bézier element, and blossom. We also generalize them to arbitrary topology AS T-splines. Linear independence in [1] is the traditional *global* linear independence, which means the blending functions are linear independent in the whole domain. Local linear independent means if a T-spline is zero in a bounded domain, then all the coefficients for the blending functions which are not zero in the domain must be zeros. It is obvious that local linear independence is a stricter requirement than global linear independence. Some numerical methods, such as *localized multi-resolution*, *linear independence for non-tensor-product domain*, rely on locally linear independence. The number of control points which contribute one Bézier element has a clear impact on the use of T-splines in iso-geometric analysis applications. Because it is possible to obtain a bound for the number of non zero entries in the mass and stiffness matrices in applications to PDEs. Blossom, introduced by Dr. Lyle Ramshaw, can be thought of as an alternative method of labeling the control points for a B-Spline curve or surface. Blossom provides a clear and insight way to understand the algorithm for B-splines [20–22]. But it is much more complex to derive the blossom formula for T-spline than those for B-splines, which are discussed in Section 5.

T-splines and the other local refinable splines, including polynomial splines over hierarchical T-meshes (PHT) [23–26], hierarchical B-splines [27,28] and LR B-splines [29], are not local linear independent. Examples for the hierarchical B-splines and LR B-splines which are linear independent but not local linear independent (also the other two properties) are straightforward. We only provide examples for T-splines and PHT. For example, in Figure 1.1a, the corresponding T-splines are always linear independent but it is not local linear independent in the grey domain because they are 21 blending functions are not zero in the domain. And PHT defined over the T-mesh in Figure 1.1b. is also not local linear independent because they are 24 basis functions (each vertex corresponds four basis functions) are not zero in the grey domain. In both examples, the corresponding vertices for these basis functions are marked with red.

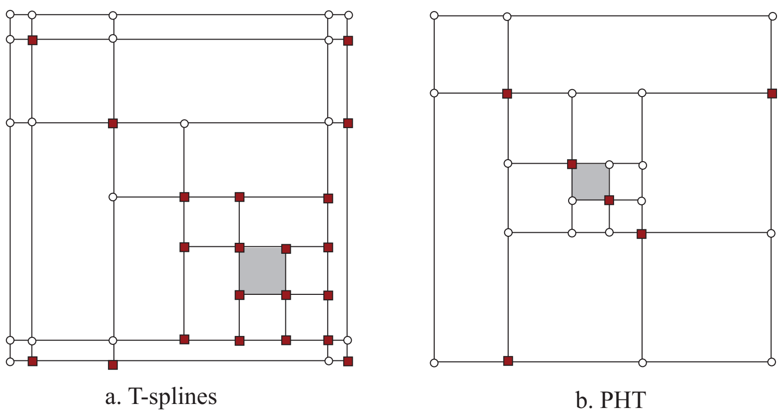

Fig. 1.1. Both T-splines and PHT are not always local linear independent even they are global linear independent.