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# A VARIATIONAL APPROACH FOR DETECTING FEATURE LINES ON MESHES*

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**Abstract**

Feature lines are fundamental shape descriptors and have been extensively applied to computer graphics, computer-aided design, image processing, and non-photorealistic rendering. This paper introduces a unified variational framework for detecting generic feature lines on polygonal meshes. The classic Mumford-Shah model is extended to surfaces. Using Γ-convergence method and discrete differential geometry, we discretize the proposed variational model to sequential coupled sparse linear systems. Through quadratic polynomials fitting, we develop a method for extracting valleys of functions defined on surfaces. Our approach provides flexible and intuitive control over the detecting procedure, and is easy to implement. Several measure functions are devised for different types of feature lines, and we apply our approach to various polygonal meshes ranging from synthetic to measured models. The experiments demonstrate both the effectiveness of our algorithms and the visual quality of results.

*Mathematics subject classification:* 65D18.
*Key words:* Feature lines, Variational approach, Polygonal meshes, The Mumford-Shah model, Discrete operators, Valleys of functions.

## 1. Introduction

In computer graphics, computer-aided design and image processing, a feature is an individual measurable heuristic property of the object being observed and is relevant for solving the computational task related to a certain application. For many applications, the feature detection is a key ingredient built on which other high-level tasks can be further performed. In this paper, the term of "feature line" is very general and may refer to sharp features (also known as creases), ridges and valleys, and contours (also known as silhouettes) and so on. These features are powerful shape descriptors which are widely used for surface reconstruction, mesh filtering, shape matching, interrogation, and non-photorealistic rendering purposes, and have gained much attention in recent years.

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### 1.1. Previous work

To detect sharp features such as creases and corners, Hoppe et al. [1] devised an energy function that captures the number of vertices in the control mesh, their connectivity, their positions, and the number and locations of sharp features. They solved the optimization problem by decomposing it into two nested subproblems: an inner continuous optimization over the control vertex positions and an outer discrete optimization over the sharp edges. Unfortunately, their method is global, complicated and time-consuming. Contrarily, Kobbelt et al. [2] recognized the sharp features directly using some simple but effective heuristic rules based on clustering of normals. Also, Ohtake et.al [3] employed the same method so as to reconstruct the sharp features of points cloud. Their methods are straightforward to implement, but suffer from the choice of the threshold parameters and the sensitivity to noise. To overcome the latter shortcoming, robust statistic technique was introduced independently by Jones et al. [4] and Fleishman et al. [5]. They estimated smoothness of a surface using different local surface predictors, which make the main difference between theirs work. Based on robust estimation of vertex positions and smoothness, the bilateral filter [6] was applied to surface denoising while preserving features. They determined a vertex on a sharp edge by the intersection of two planes. Using the forward-search paradigm, Fleishman et al. [7] proposed a robust moving least-squares technique for reconstructing a piecewise smooth surface from a potentially noisy points cloud. In their work, sharp features were handled by treating the points across sharp features as outliers. Recently, Fan et al. [8] presented a feature-preserving mesh denoising algorithm in the same spirit. They identified piecewise smooth sub-neighborhoods using a robust density-based clustering algorithm over shared nearest neighbors, and adopted second-order bilateral filters. A common character of robust statistic-based methods is that they detect and utilize sharp features locally and implicitly. The problems of representing and recovering sharp features on various types of surfaces also have been studied by many authors, such as [9-13].

Mathematically, ridges and valleys are defined as lines on a surface where the principal curvatures attain extrema along their associated principal directions. They were first investigated by Gullstrand, 1911 Nobel Laureate in Medicine, who applied the methods of physical mathematics to the study of optical images and of the refraction of light in the eye. During the last century, ridges and valleys have been extensively studied in connection with researches on classical differential geometry and singularity theory [14], analysis of medical images [15], face recognition [16], and quality control of free-form surfaces [17] etc. In the past decade, there has been considerable effort to develop methods for extracting ridge-valley structures on polygonal meshes. Based on linear interpolation and non-maximum suppression technique, Belyaev et al. [18] presented a method for ridges and valleys detection on range images and triangular meshes. In order to achieve stable results, they employed a coupled nonlinear diffusion procedure to smooth the position and normal of vertices. Using a scheme from discrete differential geometry and a smoothing filter for higher-order surface derivatives, Hildebrandt et al. [19] provided an efficient way for feature lines detection. Along a different line, Ohtake et al. [20] fitted a multi-level implicit surface to the given polygonal mesh in advance. The curvature tensor and curvature derivatives at a mesh vertex are then estimated by those at the corresponding implicit surface point. In [21], they proposed a sparse representation SLIM for approximating a set of scattered points. The SLIM allows to estimate high order surface derivatives simply, which leads to a effective feature detection technique. For the purpose of accelerating the computation of principal curvatures and their derivatives, Kim et al. [22] and Yoshizawa et al. [23] utilized modified MLS (moving least squares) and local cubic polynomial