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A NEW PSEUDOSPECTRAL METHOD ON QUADRILATERALS *

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Abstract

In this paper, we investigate a new pseudospectral method for mixed boundary value problems defined on quadrilaterals. We introduce a new Legendre-Gauss type interpolation and establish the basic approximation results, which play important roles in pseudospectral method for partial differential equations defined on quadrilaterals. We propose pseudospectral method for two model problems and prove their spectral accuracy. Numerical results demonstrate their high efficiency. The approximation results developed in this paper are also applicable to other problems defined on complex domains.

Mathematics subject classification: 65N35, 41A30, 35J05.
Key words: Legendre-Gauss type interpolation on quadrilaterals, Pseudospectral method of mixed boundary value problems.

1. Introduction

The spectral method has gained increasing popularity in scientific computations, see [1-10,19] and the references therein. The standard spectral method is available for periodic problems and problems defined on rectangular domains. Some authors developed the spectral method for triangles, quadrilaterals and polygons, see, e.g., [1, 2, 12, 15, 21]. In actual computations, the pseudospectral method is more preferable oftentimes, with which we only need to evaluate unknown functions at interpolation nodes and can deal with nonlinear terms easily. Whereas, it is not an easy job for problems defined on quadrilaterals. The main difficulty is how to design the proper basis functions, the interpolations and the related numerical quadratures on quadrilaterals. For rectangular domains, it is natural to take the products of weights of one-dimensional numerical quadratures as the weights of two-dimensional numerical quadratures, so that the two-dimensional numerical quadratures also keep the exactness. But, in the pseudospectral method for quadrilaterals, the basis functions are not polynomials generally, and so such exactness is no longer valid for them. Moreover, even if the exactness holds for the basis functions, it might still fail for their derivatives, since the derivatives usually do not belong to the same finite-dimensional sets as the basis functions themselves. Therefore, the discrete inner products and numerical quadratures appearing in the pseudospectral schemes are not equivalent to the corresponding continuous terms involved in the weak forms of considered

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problems. This fact often destroys the spectral accuracy. Recently, the authors [13] provided a pseudospectral method for quadrilaterals. But, it was successfully applied to numerical solutions of parabolic equations on quadrilaterals.

In this paper, we investigate the pseudospectral method for mixed boundary value problems defined on quadrilaterals. The next section is for preliminaries. In Sections 3 and 4, we introduce the new orthogonal approximation and the new Legendre-Gauss type interpolation on quadrilaterals respectively. We establish the basic approximation results, which play important roles in the pseudospectral method for partial differential equations defined on quadrilaterals. In Section 5, we provide pseudospectral schemes for two model problems with the convergence analysis and some numerical results indicating their high accuracy. The final section is for concluding remarks.

2. Preliminaries

Let Ω be a convex quadrilateral with the edges L_j, the vertices $Q_j = (x_j, y_j)$, and the angles $\theta_j,\ 1 \le j \le 4$, see Figure 1. We make the variable transformation:

$$x = a_0 + a_1\xi + a_2\eta + a_3\xi\eta, \qquad y = b_0 + b_1\xi + b_2\eta + b_3\xi\eta \tag{2.1}$$

where

$$\begin{array}{llll} a_0 & = \frac{1}{4}(x_1 + x_2 + x_3 + x_4), & b_0 = \frac{1}{4}(y_1 + y_2 + y_3 + y_4), \\ a_1 & = \frac{1}{4}(-x_1 + x_2 + x_3 - x_4), & b_1 = \frac{1}{4}(-y_1 + y_2 + y_3 - y_4), \\ a_2 & = \frac{1}{4}(-x_1 - x_2 + x_3 + x_4), & b_2 = \frac{1}{4}(-y_1 - y_2 + y_3 + y_4), \\ a_3 & = \frac{1}{4}(x_1 - x_2 + x_3 - x_4), & b_3 = \frac{1}{4}(y_1 - y_2 + y_3 - y_4). \end{array} \tag{2.2}$$

Then Ω is changed to the reference square $S = \{(\xi, \eta) \mid -1 < \xi, \eta < 1\}$.

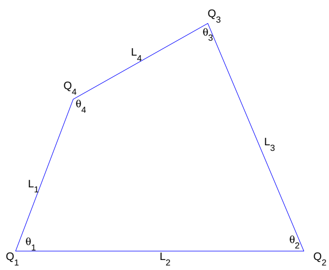

Fig. 2.1. Quadrilateral Ω.

For simplicity, we denote $\dfrac{\partial x}{\partial \xi}$ by $\partial_\xi x$, etc.. The Jacobi matrix of transformation (2.1) is

$$M_\Omega = \begin{pmatrix} \partial_\xi x & \partial_\xi y \\ \partial_\eta x & \partial_\eta y \end{pmatrix} = \begin{pmatrix} a_1 + a_3\eta & b_1 + b_3\eta \\ a_2 + a_3\xi & b_2 + b_3\xi \end{pmatrix}.$$

Its Jacobian determinant is

$$J_\Omega(\xi, \eta) = \begin{vmatrix} a_1 + a_3\eta & b_1 + b_3\eta \\ a_2 + a_3\xi & b_2 + b_3\xi \end{vmatrix} = d_0 + d_1\xi + d_2\eta \tag{2.3}$$

where

$$d_0 = a_1 b_2 - a_2 b_1, \quad d_1 = a_1 b_3 - a_3 b_1, \quad d_2 = a_3 b_2 - a_2 b_3. \tag{2.4}$$