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TOWARDS BEAM HARDENING CORRECTION FOR POLYCHROMATIC X-RAY CT*

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Abstract

High-attenuation object-induced streaking and shadow artifacts in computerized tomography (CT) are somewhat connected to the misfit of the X-ray projection data to the range space of the Radon transform. This misfit is mainly due to the beam hardening factor of the projection data which is unavoidable for polychromatic sources. The major difficulty in dealing with the beam hardening-induced streaking and shadow artifacts comes from its highly nonlinear nature depending on geometries of high attenuation objects. In this work, we investigate the mathematical characteristics of those streaking and shadow artifacts from the structure of the projection data. We also proposed a metal artifacts reduction method by incorporating the recent technique of the nonlinear beam-hardening corrector. Numerical simulations show that the proposed method effectively alleviates the streaking artifacts without changing the background images.

Mathematics subject classification: 65N38, 65N30.
Key words: Computed tomography, Metal artifact reduction, Beam hardening, Radon transform.

1. Introduction

Recently, a significant portion of patients taking CT scans contains high-attenuation objects, due to the rapidly increased number of implanted prostheses associated with a rapidly aging population. Since high-attenuation objects in CT scan may induce serious streaking and shadow artifacts, resulting in loss of information on the region adjacent to high-attenuation objects, there have been a great demand for effective metal artifact reduction (MAR) without affecting important features in CT images. High-attenuation objects causing serious streaking and shadow artifacts include hip replacements, dental fillings, surgical clips, and pacemaker.

The most commonly used CT reconstruction algorithm is the filtered back-projection (FBP) algorithm [6], which is based on the assumption that the projection data is contained in the range space of the Radon transform. However, in the presence of metallic objects, the projection data may not be located near the range space of the Radon transform as shown in Fig. 2.1. It

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seems that the degree of the discrepancy is somewhat connected to the streaking and shadow artifacts in CT images reconstructed by least square-based schemes, which seek to minimize the difference between the projection data and the Radon transform of an image function.

Over the past few decades, numerous researches on MAR have been conducted to deal with these metallic object-induced streaking and shadow artifacts. The dual-energy CT (DECT) [2] can provide the virtual monochromatic CT image (which are less affected by the metal artifacts due to beam hardening effects) using two non-linear projection data generated at two different energies. However, DECT requires a additional dose of radiation compared with single-energy CT [8]. Most of existing MAR methods can be classified into statistical iterative reconstruction methods [9,10,18,22,32], data completion/inpainting-based methods [1,4,13,15,19,23,28], and hybrid methods combining the aforementioned methods [16]. However, these existing methods have their own limitations; the iterative reconstruction techniques requires prior knowledge of energy-dependent attenuation coefficients of the materials to be imaged and the incident X-ray spectrum [30], and the use of inpainting methods may introduce new artifacts [19,20]. It would be desirable to develop a novel method of extracting streaking and shadow artifacts without affecting intact anatomical images.

In this paper, we shall limit ourselves on metal streaking and shadow artifacts induced by beam hardening, although there are several other causes of streak artifacts such as scatter, nonlinear partial volume effects, photon starvation, motion, and edge effects [5,19]. The beam hardening is brought from the inherent polychromatic nature of medical X-ray source. Metallic object-associated beam harding causes a severe discrepancy between X-ray projection data and the sinogram space, the range space of the Radon transform. We investigate how the beam hardening-induced discrepancy is linked to the streaking and shadow artifacts. Given a projection data P, let ΠP denote its orthogonal projection onto the sinogram space, as shown in Fig. 2.1. We observed that the degree of the discrepancy $\Pi P - P$ is linked to the streaking and shadow artifacts. From this observation, we conjecture the followings: If $\Pi P - P = 0$, there is no artifacts except cupping artifacts. On the other hand, if the L^2 norm of $\Pi P - P$ is not small, any least square solutions or FBP may not provide acceptable images. A rationale for this observation is provided in Section 2.

In Section 3, we propose a reconstruction method using the recent MAR technique of the geometric corrector invented by Park *et al.* [26], which can handle the inconsistency of projection data P. The geometric corrector is a function of metal geometries and a control parameter associated with all energy-dependent factors including attenuation coefficients and the spectrum of X-ray source. The novelty of the geometric corrector [26] is to extract the metal-induced streaking and shadow artifacts selectively without affecting intact anatomical images when prior knowledge about the shapes of metallic object is available. The proposed method takes advantage of this geometric corrector to compute the parameter and the target image (beam hardening-free image) simultaneously. In addition, to suppress the noise artifacts, total-variation regularization [29] is imposed on a proposed model, and this model is efficiently solved by the augmented Lagrangian method [11,27]. We conducted numerical simulations to evaluate the performance of the proposed method.

2. Method

Let Ω be a cross-sectional slice to be imaged. Let $f(\boldsymbol{x}, E)$ be the attenuation coefficient at point $\boldsymbol{x} = (x_1, x_2) \in \Omega$ and energy level E. Throughout this section, we assume that high