

## ROBUST FRAME BASED X-RAY CT RECONSTRUCTION\*

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### Abstract

As X-ray computed tomography (CT) is widely used in diagnosis and radiotherapy, it is important to reduce the radiation dose as low as reasonably achievable. For this purpose, one may use the TV based methods or wavelet frame based methods to reconstruct high quality images from reduced number of projections. Furthermore, by using the interior tomography scheme which only illuminates a region-of-interest (ROI), one can save more radiation dose. In this paper, a robust wavelet frame regularization based model is proposed for both global reconstruction and interior tomography. The model can help to reduce the errors caused by mismatch of the huge sparse projection matrix. A three-system decomposition scheme is applied to decompose the reconstructed images into three different parts: cartoon, artifacts and noise. Therefore, by discarding the estimated artifacts and noise parts, the reconstructed images can be obtained with less noise and artifacts. Similar to other frame based image restoration models, the model can be efficiently solved by the split Bregman algorithm. Numerical simulations show that the proposed model outperforms the FBP and SART+TV methods in terms of preservation of sharp edges, mean structural similarity (SSIM), contrast-to-noise ratio, relative error and correlations. For example, for real sheep lung reconstruction, the proposed method can reach the mean structural similarity as high as 0.75 using only 100 projections while the FBP and the SART+TV methods need more than 200 projections. Additionally, the proposed robust method is applicable for interior and exterior tomography with better performance compared to the FBP and the SART+TV methods.

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## 1. Introduction

In the clinical applications of X-ray Computed Tomography (CT), it is important to reduce the X-ray dose while preserving the quality of CT image reconstruction. The X-ray CT reconstruction problem can be essentially represented as a linear inverse problem:

$$Pu = f, \quad (1.1)$$

where  $P \in \mathbb{R}^{m \times n}$  is a measurement matrix representing the collection of discrete line integrations at different projection angles and along different beamlets,  $u \in \mathbb{R}^n$  is a vector rearranging from a 2 dimensional image and  $f \in \mathbb{R}^m$  is the measurement of  $u$ . The CT reconstruction is to recover the image  $u$  from a given  $P$  and  $f$  [26]. Because  $P$  is determined by the direction and location of the available beamlets, the matrix  $P$  can be approximately generated by the information from the X-ray projection geometry. However, due to the mechanical error, beam hardening, finite source and detector cell effects, and other factors, the actual measurement  $f$  does not equal to  $Pu$ . In fact, the reconstruction problem can be redefined as:

$$(P + P_\delta)u = f + \epsilon, \quad (1.2)$$

where  $P_\delta$  represents the model mismatch part of the projection matrix  $P$  caused by all the possible factors,  $\epsilon$  is the additive noise. A difficulty of solving problems (1.1) and (1.2) is that the linear system will become ill-posed if we decrease the projection number or detector cell number for dose reduction. For example, the interior tomography [33, 34] and exterior tomography are to reconstruct a region of interest (ROI) only from the measured X-rays passing through this ROI and aided by some prior information. Appropriate application of interior and exterior tomography can reduce the X-ray dose to the patients. Fig. 1.1 shows the sinogram for full CT imaging, interior tomography and exterior tomography, the ROI correspond to the pixels whose projection lines in all angles are available. Each sinogram can be regarded as the reshape of input  $f$ , where the columns represent different projection views and the rows represent different projection lines in each view. Therefore, the interior and exterior tomography can be regarded as special CT reconstruction with incomplete Radon domain measurement.

As a result, in (1.2), the matrices  $P$  and  $P_\delta$  have much smaller number of rows comparing to the number of the columns. It is difficult to determine the most appropriate  $u$  from infinitely many solutions of the problem (1.1) and (1.2). Although the current state-of-the-art clinical CT scanner does not support a scan with reduced projections, image reconstruction from few-view projections has been a hot topic. The recent development of carbon-nano tube based X-ray source make it possible for fast switch for the acquisition of sparse projections. Similar to the flush gate in a camera, it is possible to build a special flush gate and install it in front of the conventional X-ray source to control the overexposure of X-ray.

Although there are some classical methods available, such as the filtered back projection (FBP) type methods [11, 16, 24, 25] and the algebraic reconstruction techniques (ART) [19], these methods usually suffer from artifacts especially when the number of projection is insufficient. To reconstruct the image  $u$  from the noisy measurement, some differential operator based regularization methods have been introduced. The total variation (TV) based method is one of the well-known regularization methods and has been proven by its application in various fields such as signal recovery and image processing [5, 28]. The TV-based model, sometimes called as Rudin-Osher-Fatemi (ROF) model [28], has been applied to 3D X-ray cone beam CT reconstruction [30, 31] and 2D CT reconstruction [23]. In the compressed sensing framework,