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HERMITE WENO SCHEMES WITH STRONG STABILITY PRESERVING MULTI-STEP TEMPORAL DISCRETIZATION METHODS FOR CONSERVATION LAWS*

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Abstract

Based on the work of Shu [SIAM J. Sci. Stat. Comput, 9 (1988), pp.1073-1084], we construct a class of high order multi-step temporal discretization procedure for finite volume Hermite weighted essential non-oscillatory (HWENO) methods to solve hyperbolic conservation laws. The key feature of the multi-step temporal discretization procedure is to use variable time step with strong stability preserving (SSP). The multi-step temporal discretization methods can make full use of computed information with HWENO spatial discretization by holding the former computational values. Extensive numerical experiments are presented to demonstrate that the finite volume HWENO schemes with multi-step discretization can achieve high order accuracy and maintain non-oscillatory properties near discontinuous region of the solution.

Mathematics subject classification: 65M06.
Key words: Multi-step temporal discretization; Hermite weighted essentially non-oscillatory scheme; Uniformly high order accuracy; Strong stability preserving; Finite volume scheme.

1. Introduction

In this paper, we construct a class of high order multi-step temporal discretization procedure for finite volume HWENO (Hermite weighted essential non-oscillatory) methods to solve hyperbolic conservation laws:

$$\begin{cases} u_t + \nabla \cdot F(u) = 0, \\ u(x,0) = u_0(x). \end{cases} \tag{1.1}$$

In recent years, WENO (weighted essentially non-oscillatory) schemes have been designed as a class of high order finite volume or finite difference schemes to solve hyperbolic conservation laws with the property of maintaining both uniform high order accuracy and an essentially

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non-oscillatory shock transition. WENO schemes were designed based on ENO (essentially non-oscillatory) schemes in [8,21,22]. In [12], the first WENO scheme was proposed for a third-order finite volume version in one space dimension. Third and fifth-order finite difference WENO schemes in multi-space dimensions were constructed in [9], with a general framework for the design of the smoothness indicators and nonlinear weights. Higher order finite difference WENO schemes were constructed in [1], and finite volume WENO on unstructured and structured meshes were constructed in [4, 7, 11, 13, 19]. WENO improves upon ENO in robustness, better smoothness of fluxes, better steady state convergence, better provable convergence properties, and more efficiency.

Finite volume Hermite WENO schemes were proposed in [3,14,15,26,27] and also successfully applied in solving Hamilton-Jocobi equation [16, 17, 28, 29]. Hermite WENO schemes improve the dissipation properties of a WENO scheme due to reducing its stencil width. In fact, the compactness of a numerical stencil owes many advantages: the first, boundary conditions and complex geometries are easier to solve; the second, for the same formal accuracy, compact stencils are known to exhibit more resolution of the smaller scales by improving the dispersive and the dissipative properties of the numerical scheme, [10, 23].

WENO/HWENO is a spatial discretization procedure, namely, it is a procedure to approximate the spatial derivative terms in (1.1). The time derivative term there must also be discretized. In [25], strong stability preserving (SSP) high order temporal discretizations keeping the maximum principle are used. The SSP temporal discretization methods were first proposed in [20, 21], and were termed TVD (Total Variation Diminishing) temporal discretizations because the method of lines is about solving an ordinary differential equation (ODE) in time and its Euler forward version satisfy the total variation diminishing property when applied to scalar one dimensional nonlinear hyperbolic conservation laws. A class of second to fifth order SSP Runge-Kutta temporal discretizations was developed in [21]. Shu proposed a class of first order Runge-Kutta temporal discretization which have large CFL number, as well as a class of high order multi-step SSP methods in [20]. In [5], Gottlieb and Shu performed a systematic study of Runge-Kutta SSP methods, showing the optimal two stage second-order and three stage third-order SSP Runge-Kutta methods. Moreover, they proved the non-existence of four stage fourth-order SSP Runge-Kutta methods with non-negative coefficients. In [6], Gottlieb et al. reviewed and further developed SSP Runge-Kutta and multi-step methods. The new results in [6] include the optimal explicit SSP linear Runge-Kutta methods, their application to the strong stability of coercive approximations, a systematic study of explicit SSP multi-step methods, and the study of the strong stability preserving property of implicit Runge-Kutta and multi-step methods.

Multi-step temporal discretization methods can make full use of given information with spatial discretization, however, the conventional multi-step temporal discretization which is based on equal time step is not suitable for nonlinear conservation laws, for time step is variable with fixed CFL number. In this paper, we generalize the optimal few stages multi-step methods to variable size version to solve nonlinear conservation laws.

The paper is organized as follows. A brief description of the semi-discrete finite volume Hermite WENO schemes and a class of high order SSP variable step multi-step temporal discretization methods are presented in Section 2. Numerical examples are shown in the Section 3 to demonstrate the advantages of maintaining high order accuracy, the resolution and cost effective of the constructed schemes. Finally concluding remarks are given in Section 4.