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IMPROVED ENTROPY-ULTRA-BEE SCHEME FOR THE EULER SYSTEM OF GAS DYNAMICS*

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Abstract

The Entropy-Ultra-Bee scheme was developed for the linear advection equation and extended to the Euler system of gas dynamics in [13]. It was expected that the technology be applied only to the second characteristic field of the system and the computation in the other two nonlinear fields be implemented by the Godunov scheme. However, the numerical experiments in [13] showed that the scheme, though having improved the wave resolution in the second field, produced numerical oscillations in the other two nonlinear fields. Sophisticated entropy increaser was designed to suppress the spurious oscillations by increasing the entropy when there are waves in the two nonlinear fields presented. However, the scheme is then not efficient neither robust with problem-related parameters. The purpose of this paper is to fix this problem. To this end, we first study a 3×3 linear system and apply the technology precisely to its second characteristic field while maintaining the computation in the other two fields be implemented by the Godunov scheme. We then follow the discussion for the linear system to apply the Entropy-Ultra-Bee technology to the second characteristic field of the Euler system in a linearlized field-by-field fashion to develop a modified Entropy-Ultra-Bee scheme for the system. Meanwhile a remark is given to explain the problem of the previous Entropy-Ultra-Bee scheme in [13]. A reference solution is constructed for computing the numerical entropy, which maintains the feature of the density and flats the velocity and pressure to constants. The numerical entropy is then computed as the entropy cell-average of the reference solution. Several limitations are adopted in the construction of the reference solution to further stabilize the scheme. Designed in such a way, the modified Entropy-Ultra-Bee scheme has a unified form with no problem-related parameters. Numerical experiments show that all the spurious oscillations in smooth regions are gone and the results are better than that of the previous Entropy-Ultra-Bee scheme in [13].

Mathematics subject classification: 65M06, 35L65.
Key words: Entropy-Ultra-Bee scheme, Step-reconstruction, Characteristic field, Reference solution.

1. Introduction

Recently, Li et al. [13] developed a new type of finite volume scheme for the linear advection equation and extended it to the Euler system of gas dynamics to improve the resolution of

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solution in the second characteristic field. The scheme was a combination of two different finite volume schemes: a so-called entropy-step scheme and the Ultra-Bee scheme.

The entropy-step scheme developed in §2 of [13] for the linear advection equation was a modified Godunov (upwind) scheme. The modification was that it computed conservatively both the solution and an entropy, which could be an arbitrary convex function of the solution. The numerical entropy was then used in the reconstruction of solution. The solution in each cell was reconstructed as a step function, a two-piece-constant function with the jump sitting at the cell's center, see Fig. 2.1 in the following section. Because of the conservation of the entropy, the scheme showed a second-order super-convergence for smooth solutions in numerical experiments, though the standard truncation error analysis showed that the scheme was only first-order accurate.

However, when computing discontinuous solutions, the scheme produced spurious oscillations near discontinuities, though the oscillation pattern was different from that of the schemes of the Lax-Wendroff type. To fix this problem, the authors of [13] used the TVD limiter of the Ultra-Bee scheme to control the reconstruction step in each cell. See [7], [15] or [13] for the Ultra-Bee scheme. The resulted scheme was thus a combination of the entropy-step scheme and the Ultra-Bee scheme, and we call it the Entropy-Ultra-Bee scheme for convenience. The scheme was then oscillation-free near discontinuities and showed numerically second-order super-convergence in smooth regions.

The authors of [13] then extended the Entropy-Ultra-Bee technology to the Euler system of gas dynamics in §4 of that paper. The developed Entropy-Ultra-Bee scheme computed the physical entropy along with the density, momentum and energy of the system. In each cell, the density was reconstructed, using the numerical entropy, as a step function, but the velocity and pressure were still reconstructed as constant functions. It was expected that the Entropy-Ultra-Bee technology be applied only to the second characteristic field of the system and the computation in the other two nonlinear fields be implemented by the Godunov scheme.

The developed Entropy-Ultra-Bee scheme did sharpen contact discontinuities and improve the resolution of density waves for the Euler system; however, spurious oscillations in density and entropy profiles occurred in smooth regions, especially near shocks and in rarefaction waves. This is because the numerical entropy was less calculated, which did not increase to the level of the Godunov scheme in the two nonlinear fields, see the explanation in Remark 4.1 in Subsection 4.2. A deeper reason behind is that the three characteristic fields of the Euler system are nonlinearly related with one another and none of them can be completely separated from the other two ones. A sophisticated entropy increaser was designed in [13] to suppress the spurious oscillations by increasing the entropy when there are shocks and rarefaction waves presented. However, it was not efficient neither robust with parameters that needed to be tuned from problem to problem.

The purpose of this paper is to fix this problem. To this end, we first study the case of a 3×3 linear system, since in this case the Entropy-Ultra-Bee technology can be precisely applied to the second characteristic field of the system while maintaining the computation in the other two fields implemented by the Godunov scheme. Following the discussion for the linear system, we then apply the technology to the second characteristic field of the Euler system in a linearlized field-by-field fashion, in which the density is viewed as the characteristic variable of the second field and the velocity and pressure are viewed as related to the other two nonlinear fields. The point of art is to construct a reference solution in each cell, see Subsection 4.2, which, while maintaining the feature of density, flats the velocity and pressure to constants. Then