

Preface

Special issue on “Optimization and Structured Solution”

The focus of this special issue is optimization methods for problems whose solutions have simple structures. These structures include sparsity (the solution has very few nonzero entries), low-rankness (the solution is a matrix of very low rank), consensus (the solution is a set of identical vectors), and beyond. The authors of the articles in this special issue use certain functions and constraints to ensure their solutions to have these structures. Such functions are typically nonsmooth, and such constraints involve all components of variables, thus posing a challenge to algorithm design. Classic algorithms using (sub)gradients and projections are either non-applicable or performing poorly. Therefore, the authors study new methods for better efficiency.

This collection of selected papers covers recent theoretical and numerical advances in optimization with structures, including optimization over networks, block coordinate descent for tensor completion, sparse optimization algorithms, multigrid methods, as well as procedures for semi-definite programming and Euclidean distance matrix construction. These papers are summarized as follows.

Jinshan Zeng and *Wotao Yin* develop an algorithm called ExtraPush for consensus optimization over directed networks, where the goal is to minimize the sum of a set of convex functions, one at each network node. The restriction is that the algorithm runs at each node and must rely on directional communication between the nodes to solve the global problem. Theoretical analysis and numerical experiments are presented. Their algorithm can outperform the previous subgradient-push algorithm significantly.

Yangyang Xu considers the problem of higher-order singular value decomposition from incomplete data. The problem is formulated as simultaneously completing the missing values and performing tensor decomposition. Then a fast algorithm is proposed based on block coordinate update with convergence guarantees. Numerical comparisons on face recognition and MRI image reconstruction are shown.

In the paper by *Houfeng Huang*, *Qing Ling*, *Wei Shi* and *Jinlin Wang*, the problem of collaborative resource allocation over a hybrid cloud center and edge server network is set up as an optimization model. By taking advantage of the separable structure, the alternating direction method of multipliers is applied so that the edge servers and the cloud center autonomously collaborate to compute their local optimization variables and prices of network resources.

Penghang Yin and *Jack Xin* present an iterative ℓ_1 minimization procedure for a class of concave sparse metrics for compressed sensing. Their procedure uses the difference of convex functions. They show that the proposed framework has better performance than the basis pursuit. The MRI applications verify that the proposed algorithm can faithfully recover phantom images.

Cong D. Dang, *Guanghui Lan* and *Zaiwen Wen* consider smooth and nonsmooth formulations for solving linear matrix inequalities. A subgradient type method and a Nesterov type method are proposed, and their linear convergence is established. They further examine an

accelerated prox-level method without requiring the input of any problem parameters and a special case of solving a linear system of inequalities.

Qingna Li and *Houduo Qi* propose an inexact smoothing Newton method for Euclidean distance matrix optimization by taking advantage of the order information known as nonmetric MDS. Extensive numerical experiments show that the algorithm can often handle an enormous number of ordinal constraints very efficiently.

For a class of inequality constrained optimization problems in functional spaces, *Michael Ulbrich*, *Stefan Ulbrich* and *Daniela Bratzke* develop and analyze multigrid semismooth Newton methods. Superlinear local convergence is established by adding a suitable Moreau-Yosida type regularization. The analysis of a multilevel preconditioner for the semismooth Newton system is also provided.

For the minimization of a penalty function over residual functions, *Jianchao Huang*, *Zaiwen Wen* and *Xiaotao Xiao* extend the Levenberg-Marquardt framework and develop inexact variants where the inner subproblem is not solved exactly or the Jacobian is simplified. Numerical experiment shows that these methods may be more efficient than the limited-memory BFGS method.

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Zaiwen Wen	Peking University
Wotao Yin	UCLA