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MEAN SQUARE STABILITY AND DISSIPATIVITY OF SPLIT-STEP THETA METHOD FOR NONLINEAR NEUTRAL STOCHASTIC DELAY DIFFERENTIAL EQUATIONS WITH POISSON JUMPS*

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Abstract

In this paper, a split-step θ (SST) method is introduced and used to solve the nonlinear neutral stochastic differential delay equations with Poisson jumps (NSDDEwPJ). The mean square asymptotic stability of the SST method for nonlinear neutral stochastic differential equations with Poisson jumps is studied. It is proved that under the one-sided Lipschitz condition and the linear growth condition, the SST method with $\theta \in (0, 2-\sqrt{2})$ is asymptotically mean square stable for all positive step sizes, and the SST method with $\theta \in (2-\sqrt{2}, 1)$ is asymptotically mean square stable for some step sizes. It is also proved in this paper that the SST method possesses a bounded absorbing set which is independent of initial data, and the mean square dissipativity of this method is also proved.

Mathematics subject classification: 65N06, 65B99.
Key words: Neutral stochastic delay differential equations, Split-step θ method, Stability, Poisson jumps.

1. Introduction

Stochastic functional differential equations (SFDEs) play important roles in science and engineering applications, especially for systems whose evolutions in time are influenced by random forces as well as their history information. When the time delays in SFDEs are constants, they turn into stochastic delay differential equations (SDDEs). Both the theory and numerical methods for SDDEs have been well developed in the recent decades, see[1–8]. Recently many dynamical systems not only depend on the present and the past states but also involve derivatives with delays, they are described as the neutral stochastic delay differential equations (NSDDEs). Compared to the stochastic functional differential equations and the stochastic delay differential equations, the study of the neutral stochastic delay differential equations has just started. In 1981, Kolmanovskii and Myshkis [8] took the environmental disturbances into account, introduced the NSDDEs and gave their applications in chemical engineering and aeroelasticity. The analytical solutions of NSDDEs are hardly to obtain, many authors have to study

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the numerical methods for NSDDEs, Wu and Mao [9] studied the convergence of the Euler-Maruyama method for neutral stochastic functional differential equations under the one-side Lipschitz conditions and the linear growth conditions. In 2009, Zhou and Wu [10] studied the convergence of the Euler-Maruyama method for NSDDEs with Markov switching under the one-side Lipschitz conditions and the linear growth conditions. The convergence of θ method and the mean square asymptotic stability of the semi-implicit Euler method for NSDDEs were studied in [11–14]. The convergence and mean square stability of split-step θ method and split-step backward Euler method for stochastic differential equations, stochastic delay differential equations and stochastic delay integro-differential equations were studied in [15–19]. The dissipativity and mean square stability of numerical methods for neutral stochastic delay differential equations were studied by Huang [20–22] and Zong et al. [23].

In addition, stochastic delay differential equations with Poisson jumps have become very popular in modeling the phenomena arising in the fields such as economics, physics, biology, medicine, and so on. Very recently, stochastic delay equations with Poisson jumps have attracted the interest of many researchers, see, e.g.,[24–28]. To the best of our knowledge, so far no work has been reported in the literature about NSDDEwPJ and this paper will close this gap.

The aim of this paper is to study the mean square stability and dissipativity of the split-step θ method with some conditions and the step constrained for NSDDEwPJ.

The paper is organized as follows: in section 2, some stability definitions about the analytic solutions for NSDDEwPJ are introduced, some notations and preliminaries are also presented in this section. In section 3, the split-step θ method is introduced and used to solve the NSDDEwPJ, the asymptotic stability of the split-step θ method is proved. In section 4, the long time behavior of numerical solution is studied and the mean-square dissipativity result of the method is illustrated. In Section 5, some numerical experiments are given to confirm the theoretical results.

2. Mean-square asymptotic stability of analytic solution

Let $|.|$ denotes both the Euclidean norm in R^d and the trace (or Frobenius) norm in $R^{d\times l}$ (denoted by$|A|=\sqrt{\mathrm{trace}(A^{\mathrm{T}}A)}$, if A is a vector or matrix, its transpose is denoted by A^{T}. Let $\{\Omega, F, (F_t)_{t\geq 0}, \mathrm{P}\}$ define a complete probability space with a filtration$\{F_t\}_{t\geq 0}$ which is increasing and right continuous, and F_0 contain all P-null sets. Let $w(t)=(w_1(t), w_2(t), \cdots, w_l(t))^{\mathrm{T}}$ denote standard l-dimensional Brownian motion on the probability space, and $w_i(t)$is the i-element of the l-dimention vector $w(t)$. In this paper we talk about the d-dimensional NSDDEs with Poisson jumps with the following form:

$$\begin{cases} \mathrm{d}(y(t)-\Psi(y(t-\tau))) = f(t,y(t),y(t-\tau))\mathrm{d}t + g(t,y(t),y(t-\tau))\mathrm{d}\omega(t) \\ \qquad\qquad\qquad\qquad\qquad + h(t,y(t),y(t-\tau))\mathrm{d}(N(t)), \quad t\geq 0, \\ y(t)=\phi(t), \quad t\in[-\tau,0]. \end{cases} \tag{2.1}$$

where $\Psi: R^d \mapsto R^d$, $f: R_+\times R^d\times R^d \mapsto R^d$, $g: R_+\times R^d\times R^d \mapsto R^{d\times l}$ and $h: R_+\times R^d\times R^d \mapsto R^d$ are the Borel measurable functions, τ is a positive constant delay, and $\phi(t)$ is an F_0 -measurable, $\mathrm{C}[-\tau,0]; R^d)$ -valued random variable which satisfies

$$\sup_{-\tau\leq t\leq 0} \mathrm{E}[\phi^{\mathrm{T}}(t)\phi(t)] < \infty, \tag{2.2}$$

with the notation E denoting the mathematical expectation with respect to P .

The following conditions (a1) and (a2) are standard for the existence and uniqueness of the solution for (2.1):

(a_1) (The local Lipschitz condition). There exists constants K_L and $L > 0$, such that the following inequality holds,

$$\begin{aligned}&|f(t,x_1,y_1)-f(t,x_2,y_2)|^2 \vee |g(t,x_1,y_1)-g(t,x_2,y_2)|^2 \vee |h(t,x_1,y_1)-h(t,x_2,y_2)|^2\\ \leq &K_L(|x_1-x_2|^2+|y_1-y_2|^2),\end{aligned} \tag{2.3}$$

for all $|x_1| \vee |x_2| \vee |y_1| \vee |y_2| \leq L$ and $t \in R_+$, where $x_i, y_i \in R^d,\ i = 1,\ 2,$ and $a \vee b$ represents $\max\{a, b\}$, $a \wedge b$ represents $\min\{a, b\}$.

(a_2) (The linear growth condition). There exists a constant $K_G > 0$, such that the following inequality holds,

$$|f(t,x,y)|^2 \vee |g(t,x,y)|^2 \vee |h(t,x,y)|^2 \vee |\Psi(y)|^2 \leq K_G(1+|x|^2+|y|^2) \tag{2.4}$$

for all $(t, x, y) \in R_+ \times R^d \times R^d$.

As an especial case in Maos monograph [6, Theorem 3.1], we can easily know that under hypothesis (a_1) and (a_2), the system (2.1) has a global unique continuous solution on $t \geq -\tau$, which is denoted by $y(t)$.

Now we recall some stability concepts for the solution of (2.1).

Definition 2.1. *The trivial solution of* (2.1) *with initial data* (2.2) *is said to be pth moment asymptotically stable, if there exists a constant* $\delta_0 \geq 0$, *under the condition that* $\|\phi(t)\| < \delta_0$, *such that,*

$$\lim_{t\to\infty} \mathrm{E}[|y(t)|^p] = 0, \tag{2.5}$$

where $p \in Z^+$. *When* $p = 2$ *it is usually said to be asymptotically mean-square stable.*

Lemma 2.1. ([6]) *Assume that there exist a symmetric, positive definite* $d \times d$ *matrix* Q *and positive constants* μ_1, μ_2, *and* $\lambda \in (0, 1)$ *such that for all* $(t, x, y) \in \mathrm{R}_+ \times \mathrm{R}^d \times \mathrm{R}^d$ *and* $|\Psi(y)| \leq \lambda|y|$,

$$\begin{aligned}&(x-\Psi(y))^{\mathrm{T}}Q[f(t,x,y)+\kappa h(t,x,y)]+\frac{1}{2}trace[g^{\mathrm{T}}(t,x,y)\mathrm{Q}g(t,x,y)]\\ \leq &-\mu_1 x^{\mathrm{T}}\mathrm{Q}x+\mu_2 y^{\mathrm{T}}\mathrm{Q}y.\end{aligned} \tag{2.6}$$

holds. If conditions

$$0 < \lambda < \frac{1}{2}, \qquad \mu_1 > \frac{\mu_2}{(1-2\lambda)^2}, \tag{2.7}$$

hold, then the trivial solution of (2.1) *is asymptotically mean square stable.*

In general, we require $\lambda \neq 0$. When $\lambda = 0$, (2.1) becomes a stochastic delay differential equation. Many stability results have been studied in the literature.

By the lemma 2.1, the following result can easily be obtained.

Theorem 2.1. *.Suppose* $|\Psi(y)| \leq \lambda|y|$ *holds, and assume that there are constant* κ *and positive constants* λ_1, λ_2, K, *such that for all* $x, y \in R^d$, *the following inequality hold,*

$$\begin{cases}(x-\Psi(y))Q\left[f(t,x,y)+\kappa h(t,x,y)\right] \leq -\lambda_1 x^{\mathrm{T}}Qx+\lambda_2 y^{\mathrm{T}}\mathrm{Q}y,\\ |f(t,x,y)|^2 \vee |g(t,x,y)|^2 \vee |h(t,x,y)|^2 \leq K(|x|^2+|y|^2).\end{cases} \tag{2.8}$$

If conditions

$$0<\lambda<\frac{1}{2}, \qquad \lambda_1>\frac{1}{2}K+\frac{2\lambda_2+K}{2(1-2\lambda)^2}, \tag{2.9}$$

hold, then the trivial solution of (2.1) *is asymptotically mean square stable.*

Proof. Consider (2.8) and the inequality (2.6), we get the following inequality:

$$\begin{aligned}&(x-\Psi(y))^{\mathrm{T}}Q[f(t,x,y)+\kappa h(t,x,y)]+\frac{1}{2}\mathrm{trace}[g^{\mathrm{T}}(t,x,y)Qg(t,x,y)]\\ \leq&-\lambda_1 x^{\mathrm{T}}Qx+\lambda_2 y^{\mathrm{T}}Q+\frac{1}{2}K(x^{\mathrm{T}}Qx+y^{\mathrm{T}}Qy)\\ \leq&-(\lambda_1-\frac{1}{2}K)x^{\mathrm{T}}Qx+(\lambda_2+\frac{1}{2}K)\mu_2 y^{\mathrm{T}}Qy.\end{aligned}$$

Let $\mu_1=(\lambda_1-\frac{1}{2}K)$ and $\mu_2=(\lambda_2+\frac{1}{2}K)$. If conditions (2.9) hold, then we get that

$$\mu_1>\frac{\mu_2}{(1-2\lambda)^2}.$$

Using Lemma 2.1, we can easily prove that the trivial solution of (2.1) is asymptotically mean square stable. □

3. The stability of the split-step θ method

The split-step θ method is proved to be able to keep the mean square asymptotic stability of the exact solution under the sufficient conditions of the asymptotic stability of the exact solution, so in this paper we use the split-step θ method to solve the NSDDEwPJ.

Apply the split-step θ method into problem (2.1) gives the following form:

$$\begin{cases}Y_{n+1}-\Psi Y_{n+1-m}=y_n-\Psi y_{n-m}+\theta\Delta t f(t_n+\theta\Delta t, Y_n, \bar{Y_n}),\\ y_{n+1}-\Psi y_{n+1-m}=y_n-\Psi y_{n-m}+\Delta t f(t_n+\theta\Delta t, Y_n+\bar{Y_n})\\ \qquad\qquad +g(t_n+\theta\Delta t, Y_n, \bar{Y_n})\Delta w_n+h(t_n, Y_n, \bar{Y_n})\Delta N_n,\end{cases} \tag{3.1}$$

where the stepsize $\Delta t=\frac{\tau}{m}$, m is an integer, then $\bar{Y_n}=Y_{n-m}$. y_i is an approximation to $y(t_i)$, for $t_i=i\Delta t$, $i=1,2,\cdots$, $Y_k=y_k=\phi(k\Delta t)$ for $k=-m,-m+1,\cdots,0$. $\theta\in[0,1]$ is a fixed parameter, and $\Delta w_k:=w((k+1)\Delta t)-w(k\Delta t)$ is the Brownian increment, $\Delta N_j:=N(j+1)\Delta t)-N(j\Delta t)$ is d -dimensional Poisson process with intensity κ.

When $\theta=0$ the split-step θ method is simplified into the split-step forward Euler method and $\theta=1$ the split-step θ method is simplified into the split-step backward Euler method. They were discussed for stochastic differential equations in [16–19]. In order to consider the stability property of the scheme (3.1) we should give some convergence and stability concepts for numerical methods firstly.

Definition 3.1. *A numerical method is said to be asymptotically mean square convergent if for any initial data $\phi(t)$, the numerical solution y_n produced by the method satisfies*

$$\lim_{n\to\infty} \mathrm{E}|y(t_n) - y_n|^2 = 0$$

where $y(t_n)$ is the exact solution at $t = t_n$.

Remark 3.1. For the asymptotically mean square convergence of split-step θ method for stochastic delay differential equation with Poisson jump and neutral stochastic delay differential equation with Poisson jump, it had been studied and proved in [27, 28].

Definition 3.2. *For a given stepsize Δt, a numerical method is said to be asymptotically mean square stable if for any initial data $\phi(t)$, there exist a symmetric, positive definite $d \times d$ matrix Q, such that the numerical solution y_n produced by the method, satisfies*

$$\lim_{n\to\infty} E[y_n^{\mathrm{T}} Q y_n] = 0.$$

Theorem 3.1. *Assume system* (2.1) *satisfies* (2.6) *with* $[-\mu_1+\mu_2+|\kappa|(1+2|\kappa|)(K_3+K_4) < 0]$. *If we further assume that there exist constants K_1, K_2, K_3 and K_4, such that*

$$f^{\mathrm{T}}(t,x,y)Qf(t,x,y) \le K_1 x^{\mathrm{T}} Q x + K_2 y^{\mathrm{T}} Q y, \tag{3.2}$$

$$h^{\mathrm{T}}(t,x,y)Qh(t,x,y) \le K_3 x^{\mathrm{T}} Q x + K_4 y^{\mathrm{T}} Q y, \tag{3.3}$$

then for $\theta \in [2-\sqrt{2}, 1)$, there exist a constant Δt_0 depending on θ, such that the SST method (3.1) *is asymptotically mean square stable for $\Delta t \in (0, \Delta t_0)$; For $0 < \theta < 2-\sqrt{2}$ the SST method* (3.1) *is asymptotically mean square stable for all step size.*

Proof. From (3.1) it follows that

$$\begin{aligned}
&(y_{n+1} - \Psi y_{n+1-m})^{\mathrm{T}} Q (y_{n+1} - \Psi y_{n+1-m}) && (3.4)\\
=&(y_n - \Psi y_{n-m})^{\mathrm{T}} Q (y_n - \Psi y_{n-m}) + \Delta t^2 f^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q f(t_n, Y_n, \bar{Y_n})\\
&+ \Delta w_n^{\mathrm{T}} g^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q g(t_n, Y_n, \bar{Y_n}) \Delta w_n + \Delta N_n^{\mathrm{T}} h^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q h(t_n, Y_n, \bar{Y_n}) \Delta N_n\\
&+ 2(y_n - \Psi y_{n-m})^{\mathrm{T}} \Delta t Q f(t_n, Y_n, \bar{Y_n}) + 2(y_n - \Psi y_{n-m})^{\mathrm{T}} Q g(t_n, Y_n, \bar{Y_n}) \Delta w_n\\
&+ 2(y_n - \Psi y_{n-m})^{\mathrm{T}} Q h(t_n, Y_n, \bar{Y_n}) \Delta N_n + 2\Delta t f^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q g(t_n, Y_n, \bar{Y_n}) \Delta w_n\\
&+ 2\Delta t f^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q h(t_n, Y_n, \bar{Y_n}) \Delta N_n + \Delta w_n^{\mathrm{T}} g^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q h(t_n, Y_n, \bar{Y_n}) \Delta N_n.
\end{aligned} \tag{3.5}$$

Since $w(t) = (w_1(t), w_2(t), \cdots, w_l(t))^{\mathrm{T}}$ is a standard l -dimensional Brownian motion and $\Delta N_j := N((j+1)\Delta t) - N(j\Delta t)$ is Poisson distribution with the parameter is λ , we have that

$$\begin{aligned}
&\mathrm{E}(\Delta w_i) = 0,\ \mathrm{E}[(\Delta w_i)^2] = \Delta t,\ \mathrm{E}(\Delta N_i) = \kappa \Delta t,\ \mathrm{E}[(\Delta N_i)^2] = \kappa \Delta t(1+\kappa \Delta t),\\
&\mathrm{E}[\Delta w_n^{\mathrm{T}} g^{\mathrm{T}}(t_n + \theta \Delta t, Y_n, \bar{Y_n}) Q g(t_n + \theta \Delta t, Y_n, \bar{Y_n}) \Delta w_n]\\
=&\Delta t \mathrm{E}[\mathrm{trace} g^{\mathrm{T}}(t_n + \theta \Delta t, Y_n, \bar{Y_n}) Q g(t_n + \theta \Delta t, Y_n, \bar{Y_n})], && (3.6)\\
&\mathrm{E}[\Delta N_n^{\mathrm{T}} h^{\mathrm{T}}(t_n + \theta \Delta t, Y_n, \bar{Y_n}) Q h(t_n + \theta \Delta t, Y_n, \bar{Y_n}) \Delta N_n]\\
=&(\kappa \Delta t)(1+\kappa \Delta t) \mathrm{E}[h^{\mathrm{T}}(t_n + \theta \Delta t, Y_n, \bar{Y_n}) Q h(t_n + \theta \Delta t, Y_n, \bar{Y_n})]. && (3.7)
\end{aligned}$$

Let $X_n = Y_n - \Psi Y_{n-m}$ and $x_n = y_n - \Psi y_{n-m}, n = 0, 1, \cdots$. Substituting the designation into (3.6) and then taking expectation on both sides, yield

$$
\begin{aligned}
&\mathrm{E}[x_{n+1}^{\mathrm{T}} Q x_{n+1}]\\
\le&\mathrm{E}[x_n^{\mathrm{T}} Q x_n] + \Delta t^2 \mathrm{E}[f^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q f(t_n, Y_n, \bar{Y_n})] + \Delta t \mathrm{E}[\mathrm{trace} g^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q g(t_n, Y_n, \bar{Y_n})]\\
&+ \kappa\Delta t(1+\kappa\Delta t)\mathrm{E}[h^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q h(t_n, Y_n, \bar{Y_n})] + 2\Delta t \mathrm{E}[x_n^{\mathrm{T}} Q f(t_n, Y_n, \bar{Y_n})]\\
&+ 2\kappa\Delta t \mathrm{E}[x_n^{\mathrm{T}} Q h(t_n, Y_n, \bar{Y_n})] + 2\Delta t\kappa\Delta t \mathrm{E}[f(t_n, Y_n, \bar{Y_n})^{\mathrm{T}} Q h(t_n, Y_n, \bar{Y_n})]\\
\le&\mathrm{E}[x_n^{\mathrm{T}} Q x_n] + \Delta t^2 \mathrm{E}[f^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q f(t_n, Y_n, \bar{Y_n})] + \Delta t \mathrm{E}[\mathrm{trace} g^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q g(t_n, Y_n, \bar{Y_n})]\\
&+ \kappa\Delta t(1+\kappa\Delta t)\mathrm{E}[h^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q h(t_n, Y_n, \bar{Y_n})] + 2\Delta t \mathrm{E}[(X_n - \theta\Delta t f(t_n+\theta\delta t, Y_n, \bar{Y_n}))^{\mathrm{T}} Q f(t_n, Y_n, \bar{Y_n})]\\
&+ 2\kappa\Delta t \mathrm{E}[(X_n - \theta\Delta t f(t_n+\theta\delta t, Y_n, \bar{Y_n}))^{\mathrm{T}} Q h(t_n, Y_n, \bar{Y_n})] + 2\Delta t\kappa\Delta t \mathrm{E}[f(t_n, Y_n, \bar{Y_n}))^{\mathrm{T}} Q h(t_n, Y_n, \bar{Y_n})]\\
\le&\mathrm{E}[x_n^{\mathrm{T}} Q x_n] + (1-2\theta)\Delta t^2 \mathrm{E}[f^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q f(t_n, Y_n, \bar{Y_n})] + \Delta t \mathrm{E}[\mathrm{trace} g^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q g(t_n, Y_n, \bar{Y_n})]\\
&+ \kappa\Delta t(1+\kappa\Delta t)\mathrm{E}[h^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q h(t_n, Y_n, \bar{Y_n})] + 2\Delta t \mathrm{E}[(X_n^{\mathrm{T}} Q f(t_n, Y_n, \bar{Y_n})]\\
&+ 2\kappa\Delta t \mathrm{E}[X_n^{\mathrm{T}} Q h(t_n, Y_n, \bar{Y_n})] + 2(1-\theta)\Delta t\kappa\Delta t \mathrm{E}[f^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q h(t_n, Y_n, \bar{Y_n})]\\
\le&\mathrm{E}[x_n^{\mathrm{T}} Q x_n] + [(\theta-2)^2 - 2]\Delta t^2 \mathrm{E}[f^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q f(t_n, Y_n, \bar{Y_n})]\\
&+ \Delta t \mathrm{E}[\mathrm{trace} g^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q g(t_n, Y_n, \bar{Y_n})] + \kappa\Delta t(1+2\kappa\Delta t)\mathrm{E}[h^{\mathrm{T}}(t_n, Y_n, \bar{Y_n}) Q h(t_n, Y_n, \bar{Y_n})]\\
&+ 2\Delta t \mathrm{E}[X_n^{\mathrm{T}} Q f(t_n, Y_n, \bar{Y_n})] + 2\kappa\Delta t \mathrm{E}[X_n^{\mathrm{T}} Q h(t_n, Y_n, \bar{Y_n})].
\end{aligned}
$$

which combined with (2.6), gives

$$
\begin{aligned}
\mathrm{E}[x_{n+1}^{\mathrm{T}} Q x_{n+1}] \le&\mathrm{E}[x_n^{\mathrm{T}} Q x_n] + 2\Delta t \mathrm{E}(-\mu_1 Y_n^{\mathrm{T}} Q Y_n + \mu_2 \bar{Y_n}^{\mathrm{T}} Q \bar{Y_n})\\
&+ [(\theta-2)^2 - 2]\Delta t^2 \mathrm{E}[f^{\mathrm{T}}(t_n+\theta\Delta t, Y_n, \bar{Y_n}) Q f(t_n+\theta\Delta t, Y_n, \bar{Y_n})]\\
&+ |\kappa|\Delta t(1+2|\kappa|\Delta t)\mathrm{E}(K_3 Y_n^{\mathrm{T}} Q Y_n + K_4 \bar{Y_n}^{\mathrm{T}} Q \bar{Y_n}).
\end{aligned}
$$

In the case of that $0 < \theta < 2-\sqrt{2}$, using $\Delta t f(t_n+\theta\Delta t, Y_n, \bar{Y_n}) = \frac{1}{\theta}(X_n - x_n)$, and

$$
\begin{aligned}
2X_n^{\mathrm{T}} Q x_n \le&\frac{2-(\theta-2)^2 - (-\mu_1+\mu_2+|\kappa|(1+2|\kappa|\Delta t)(K_3+K_4))\Delta t\theta^2}{2-(\theta-2)^2} X_n^{\mathrm{T}} Q X_n\\
&+ \frac{2-(\theta-2)^2}{2-(\theta-2)^2 - (-\mu_1+\mu_2+|\kappa|(1+2|\kappa|\Delta t)(K_3+K_4))\Delta t\theta^2} x_n^{\mathrm{T}} Q x_n,
\end{aligned}
\tag{3.8}
$$

then we have

$$
\begin{aligned}
&\mathrm{E}[x_{n+1}^{\mathrm{T}} Q x_{n+1}]\\
\le&\left(1 + \frac{(2-(\theta-2)^2)(-\mu_1+\mu_2+|\kappa|(1+2|\kappa|\Delta t)(K_3+K_4))\Delta t}{2-(\theta-2)^2 - (-\mu_1+\mu_2+|\kappa|(1+2|\kappa|\Delta t)(K_3+K_4))\Delta t\theta^2}\right)\mathrm{E}[x_n^{\mathrm{T}} Q x_n]\\
&+(|\kappa|(1+2|\kappa|\Delta t)K_3 - 2\mu_1)\Delta t \mathrm{E}[Y_n^{\mathrm{T}} Q Y_n] + (|\kappa|(1+2|\kappa|\Delta t)K_4 + 2\mu_2)\Delta t \mathrm{E}[\bar{Y_n}^{\mathrm{T}} Q \bar{Y_n}]
\end{aligned}
\tag{3.9}
$$

Let

$$
\begin{aligned}
k = \max\Bigg\{&1 + \frac{(2-(\theta-2)^2)(-\mu_1+\mu_2+|\kappa|(1+2|\kappa|\Delta t)(K_3+K_4))\Delta t}{2-(\theta-2)^2 - (-\mu_1+\mu_2+|\kappa|(1+2|\kappa|\Delta t)(K_3+K_4))\Delta t\theta^2},\\
&\left(\frac{|\kappa|(1+2|\kappa|\Delta t)K_4 + 2\mu_2}{2\mu_1 - |\kappa|(1+2|\kappa|\Delta t)K_3}\right)^{\frac{1}{m}}\Bigg\}.
\end{aligned}
\tag{3.10}
$$

We can deduce that $0 < k < 1$. By induction, the following results is obtained from (3.9)

$$\begin{aligned}&\mathrm{E}[x_{n+1}^{\mathrm{T}} Q x_{n+1}]\\ \leq& k^{n+1}\mathrm{E}[x_0^{\mathrm{T}} Q x_0] + (|\kappa|(1+2|\kappa|\Delta t)K_3 - 2\mu_1)\Delta t \sum_{j=0}^{n} k^{n-j}\mathrm{E}[\bar{Y_j}^{\mathrm{T}} Q \bar{Y_j}]\\ &+ (|\kappa|(1+2|\kappa|\Delta t)K_4 + 2\mu_2)\Delta t \sum_{j=0}^{n} k^{n-j}\mathrm{E}[\bar{Y_j}^{\mathrm{T}} Q \bar{Y_j}].\end{aligned} \tag{3.11}$$

Follow (3.1) it gives the following inequality

$$\begin{aligned}&\sum_{j=0}^{n} k^{n-j}\mathrm{E}[\bar{Y_j}^{\mathrm{T}} Q \bar{Y_j}]\\ \leq& mk^{n-m+1} \max_{-m\leq j\leq -1} \mathrm{E}[Y_j^{\mathrm{T}} Q Y_j] + k^{-m} \sum_{j=0}^{n-m+1} k^{n-j}\mathrm{E}[Y_j^{\mathrm{T}} Q Y_j].\end{aligned} \tag{3.12}$$

Therefore,

$$\begin{aligned}&\mathrm{E}[x_{n+1}^{\mathrm{T}} Q x_{n+1}]\\ \leq& k^{n+1}(\mathrm{E}[x_0^{\mathrm{T}} Q x_0] + \tau(|\kappa|(1+2|\kappa|\Delta t)K_4 + 2\mu_2)k^{-m} \max_{-m\leq j\leq -1} \mathrm{E}[\bar{Y_j}^{\mathrm{T}} Q \bar{Y_j}])\\ &+ \left(|\kappa|(1+2|\kappa|\Delta t)K_3 - 2\mu_1 + (|\kappa|(1+2|\kappa|\Delta t)K_4 + 2\mu_2)k^{-m}\right)\Delta t \sum_{j=0}^{n} k^{n-j}\mathrm{E}[Y_j^{\mathrm{T}} Q Y_j].\end{aligned} \tag{3.13}$$

It can be deduced from (3.12) and (3.13) that

$$|\kappa|(1+2|\kappa|\Delta t)K_3 - 2\mu_1 + (|\kappa|(1+2|\kappa|\Delta t)K_4 + 2\mu_2)k^{-m} \leq 0,$$

so, we can have the following inequality

$$\begin{aligned}&\mathrm{E}[x_{n+1}^{\mathrm{T}} Q x_{n+1}]\\ \leq& k^{n+1}(\mathrm{E}[x_0^{\mathrm{T}} Q x_0] + \tau(|\ \kappa|(1+2|\kappa|\Delta t)K_4 + 2\mu_2)k^{-m} \max_{-m\leq j\leq -1} \mathrm{E}[Y_j^{\mathrm{T}} Q Y_j]).\end{aligned}$$

On the other hand, we know that

$$\|y_{n+1}\| = \|y_{n+1} - \Psi y_{n+1-m} + \Psi y_{n+1-m}\| \leq \|x_{n+1}\| + \|\Psi y_{n+1-m}\|,$$

then we get

$$\mathrm{E}[y_{n+1}^{\mathrm{T}} Q y_{n+1}] \leq 2\mathrm{E}[x_{n+1}^{\mathrm{T}} Q x_{n+1}] + 2\lambda^2\mathrm{E}[y_{n+1-m}^{\mathrm{T}} Q y_{n+1-m}]. \tag{3.14}$$

Define

$$\varepsilon_0 = k^{n+1}(\mathrm{E}[x_0^{\mathrm{T}} Q x_0] + \tau(|\kappa|(1+2|\kappa|\Delta t)K_4 + 2\mu_2)k^{-m} \max_{-m\leq j\leq -1} \mathrm{E}[Y_j^{\mathrm{T}} Q Y_j]).$$

The following inequality can be deduced from (3.14)

$$\mathrm{E}[y_{n+1}^{\mathrm{T}} Q y_{n+1}] \leq \frac{2}{1-2\lambda^2}\varepsilon_0 + (2\lambda^2)^{\lfloor \frac{n}{m} \rfloor + 1} \max_{-m\leq j\leq -1} \mathrm{E}[Y_j^{\mathrm{T}} Q Y_j].$$

Therefore,

$$\lim_{n\to\infty} \mathrm{E}(y_n^{\mathrm{T}} Q y_n) = 0,$$

which implies that the method is asymptotically mean square stable.

For the case that $\theta \in [2-\sqrt{2}, 1)$, with the hypothesis (3.2), (3.3), and (3.8), we can obtain the following inequality

$$\begin{aligned}&\mathrm{E}[x_{n+1}^{\mathrm{T}} Q x_{n+1}]\\ \leq&\mathrm{E}[x_n^{\mathrm{T}} Q x_n] + \big([(\theta-2)^2-2]\Delta t K_1 - 2\mu_1 + |\kappa|(1+2|\kappa|\Delta t)K_3\big)\,\delta t \mathrm{E}[Y_n^{\mathrm{T}} Q Y_n]\\ &+\big([(\theta-2)^2-2]\Delta t K_2 - 2\mu_2 + |\kappa|(1+2|\kappa|\Delta t)K_4\big)\,\Delta t \mathrm{E}[\bar{Y_n}^{\mathrm{T}} Q \bar{Y_n}].\end{aligned}$$

A combination of (3.1), (3.2) and (3.3) gives

$$[x_n^{\mathrm{T}} Q x_n] \leq L_1 Y_n^{\mathrm{T}} Q Y_n + L_2 \bar{Y_n}^{\mathrm{T}} Q \bar{Y_n},$$

where $L_1 = (1+\theta\Delta t)(1+\theta\Delta t K_1), L_2 = (1+\theta\Delta t)\theta\Delta t K_2$. Let

$$\Delta t_0 = \min\left\{\frac{2(\mu_1-\mu_2) - |\kappa|(K_3+K_4)}{[(\theta-2)^2-2](K_1+K_2)}, 1\right\}.$$

Then for any fixed $\Delta t \in (0, \Delta t_0)$, we have

$$[(\theta-2)^2-2]\Delta t K_1 - 2\mu_1 + |\kappa|(1+2|\kappa|\Delta t)K_3 + [(\theta-2)^2-2]\Delta t K_2 + 2\mu_2 + |\kappa|(1+2|\kappa|\Delta t)K_4 < 0.$$

Consequently, there exists a small positive number ε such that

$$([(\theta-2)^2-2](K_1+K_2) + 2|\kappa|^2(K_3+K_4))\Delta t - 2(\mu_2-\mu_1) + |\kappa|(K_3+K_4) + \frac{L_1+L_2}{\Delta t}\varepsilon < 0.$$

Therefore,

$$\begin{aligned}&\mathrm{E}[x_{n+1}^{\mathrm{T}} Q x_{n+1}] \qquad (3.15)\\ \leq&(1-\varepsilon)\mathrm{E}[x_n^{\mathrm{T}} Q x_n] + \Big([(\theta-2)^2-2]\Delta t K_1 - 2\mu_1 + |\kappa|(1+2|\kappa|\Delta t\Big)K_3 + \frac{L_1}{\Delta t}\varepsilon)\Delta t \mathrm{E}\\ &+\Big([(\theta-2)^2-2]\Delta t K_2 - 2\mu_2 + |\kappa|(1+2|\kappa|\Delta t\Big)K_4 + \frac{L_2}{\Delta t}\varepsilon)\Delta t \mathrm{E}[\bar{Y_n}^{\mathrm{T}} Q \bar{Y_n}].\end{aligned}$$

Let

$$\widetilde{k} = \max\left\{1-\varepsilon, \left(\frac{[(\theta-2)^2-2]\Delta t K_2 - 2\mu_2 + |\kappa|(1+2|\kappa|\Delta t)K_4 + \frac{L_2}{\Delta t}\varepsilon}{-([(\theta-2)^2-2]\Delta t K_1 - 2\mu_1 + |\kappa|(1+2|\kappa|\Delta t)K_3 + \frac{L_1}{\Delta t}\varepsilon)}\right)^{\frac{1}{m}}\right\}.$$

Then $0 < \widetilde{k} < 1$. Similarly to the derivation of the first part, the following inequality can be proved from (3.15) that

$$\mathrm{E}[x_{n+1}^{\mathrm{T}} Q x_{n+1}] \leq \Big(\mathrm{E}[x_0^{\mathrm{T}} Q x_0] + \widetilde{L}\widetilde{k}^{-m} \max_{-m\leq j\leq -1} \mathrm{E}[Y_j^{\mathrm{T}} Q Y_j]\Big),$$

where $\widetilde{L} = \tau([(\theta-2)^2-2]\Delta t K_2 + 2\mu_2 + |\kappa|(1+2|\kappa|\Delta t)K_4 + \frac{L_2}{\Delta t}\varepsilon)$. We can easily prove that when $\Delta t \in (0, \Delta t_0)$, the method is asymptotically mean square stable. The proof of theorem is completed. $\square$

Remark 3.2. For system (2.1) with $\Psi = 0$, it becomes a stochastic delay differential equation with Poisson jump, the mean square stability of the theta method has been studied in [25] and [26].

4. Mean square dissipativity

The numerical solutions long time dynamic behavior will be studied in this section. Before it, we make the following hypothesis: assume that there exist a symmetric, positive definite $d \times d$ matrix Q and positive constants μ_1 , μ_2 ,γ such that for all $(t, x, y) \in R_+ \times R^d \times R^d$, the following inequality exists

$$\begin{aligned}&(x-\Psi(y))^{\mathrm{T}}Q[f(t,x,y)+\kappa h(t,x,y)]+\frac{1}{2}\mathrm{trace}[g^{\mathrm{T}}(t,x,y)Qg(t,x,y)]\\ \le&\gamma-\mu_1 x^{\mathrm{T}}Qx+\mu_2 y^{\mathrm{T}}Qy.\end{aligned}\tag{4.1}$$

Now we state and prove some conclusions.

Definition 4.1. *Assume that system* (2.1) *satisfies the conditions of Theorem* 3.1*. The numerical method is said to be dissipative if, when the method is applied to problem* (2.1) *with constraint $\tau = mh$, there exists a constant C such that, for any initial values, there exists an n_0, depends only on initial values $\phi(t)$, such that*

$$\mathrm{E}[y_n^{\mathrm{T}}Qy_n] \le C, \quad n \ge n_0. \tag{4.2}$$

Theorem 4.1. *Assume that system* (2.1) *satisfies the conditions of Theorem* 3.1*, there exists a constant C such that for any initial values, there exists an n_0 depending only on the initial values $\phi(t)$, when $n \ge n_0$, the numerical solution y_n generated by the SST method* (3.1) *with $\theta \in [2-\sqrt{2}, 1)$, such that*

$$\mathrm{E}[y_n^{\mathrm{T}}Qy_n] \le C.$$

Proof. Consider (3.8) and (4.1), the following inequality can be obtained

$$\begin{aligned}&\mathrm{E}[x_{n+1}^{\mathrm{T}}Qx_{n+1}]\\ \le&\mathrm{E}[x_n^{\mathrm{T}}Qx_n]+2\Delta t\gamma+2\Delta t\mathrm{E}(-\mu_1 Y_n^{\mathrm{T}}QY_n+\mu_2\bar{Y_n}^{\mathrm{T}}Q\bar{Y_n})\\ &+[(\theta-2)^2-2]\Delta t^2\mathrm{E}[f^{\mathrm{T}}(t_n+\theta\Delta t,Y_n,\bar{Y_n})Qf(t_n+\theta\Delta t,Y_n,\bar{Y_n})]\\ &+|\kappa|\Delta t(1+2|\kappa|\Delta t)\mathrm{E}(K_3Y_n^{\mathrm{T}}QY_n+K_4\bar{Y_n}^{\mathrm{T}}Q\bar{Y_n}).\end{aligned}\tag{4.3}$$

We can get the following inequality the same as the derivation of (3.13),

$$\begin{aligned}&\mathrm{E}[x_{n+1}^{\mathrm{T}}Qx_{n+1}]\\ \le&k^{n+1}((\mathrm{E}[x_0^{\mathrm{T}}Qx_0]+\tau(|\kappa|(1+2|\kappa|\Delta t+2\mu_2))k^{-m}\max_{-m\le j\le -1}\mathrm{E}[Y_j^{\mathrm{T}}QY_j]))\\ &+\Big(|\kappa|(1+2|\kappa|\Delta t)K_3-2\mu_1+(|\kappa|(1+2|\kappa|\Delta t)K_4+2\mu_2)k^{-m}\Big)\Delta t\\ &\cdot\sum_{j=0}^{n-m+1}k^{n-j}\mathrm{E}[Y_j^{\mathrm{T}}QY_j]+2\Delta t\gamma\sum_{j=0}^{n}k^j\end{aligned}\tag{4.4}$$

where $0 < k < 1$ is the same as defined in (3.10). Since

$$|\kappa|(1+2|\kappa|\Delta t)K_3-2\mu_1+(|\kappa|(1+2|\kappa|\Delta t)K_4+2\mu_2)\le 0,$$

we have the following inequality

$$
\begin{aligned}
&\mathrm{E}[x_{n+1}^{\mathrm{T}}Qx_{n+1}]\\
\leq &k^{n+1}(\mathrm{E}[x_0^{\mathrm{T}}Qx_0]+\tau(|\kappa|(1+2|\kappa|\Delta t)K_4+2\mu_2)k^{-m}\max_{-m\leq j\leq -1}\mathrm{E}[Y_j^{\mathrm{T}}QY_j])+2\Delta t\gamma\sum_{j=0}^{n}k^j.
\end{aligned}
$$

Let

$$
\varepsilon_1=k^{n+1}(\mathrm{E}[x_0^{\mathrm{T}}Qx_0]+\tau(|\kappa|(1+2|\kappa|\Delta t)K_4+2\mu_2)k^{-m}\max_{-m\leq j\leq -1}\mathrm{E}[Y_j^{\mathrm{T}}QY_j])+\frac{2\Delta t\gamma}{1-k}.
$$

Then the following inequality can be deduced from (3.14)

$$
\begin{aligned}
&\mathrm{E}[y_{n+1}^{\mathrm{T}}Qy_{n+1}]\\
\leq &2\mathrm{E}[x_{n+1}^{\mathrm{T}}Qx_{n+1}]+2\lambda^2\mathrm{E}[y_{n+1-m}^{\mathrm{T}}Qy_{n+1-m}]\\
\leq &2\varepsilon_1+2\lambda^2\mathrm{E}[y_{n+1-m}^{\mathrm{T}}Qy_{n+1-m}]\leq C,
\end{aligned}
\tag{4.5}
$$

where $C=\frac{2\varepsilon_1}{1-2\lambda^2}+\varepsilon$, ε is the arbitrary small constant. The proof of the theorem is completed. □

Theorem 4.1 means that the discrete system possesses a bounded absorbing set in the sense of mean square. The numerical solution trajectory from any initial date will enter the set in a finite time and thereafter remain inside. It is called mean-square dissipativity.

Remark 4.1. For the NSDDEs, the mean square asymptotic stability and dissipativity results of numerical method had been studied in [14] and [22].

5. The numerical experiment

In this section, we will give a numerical experiment to illustrate the stability result obtained in Section 2. Consider the following nonlinear scalar neutral stochastic delay differential equation with Poisson jump

$$
\begin{cases}
\mathrm{d}(y(t)-\dfrac{1}{4}\sin(y(t-1)))=(-8y(t)+\sin(y(t-1)))\mathrm{d}t+(-y(t)+2\sin(y(t-1)))\mathrm{d}W(t)\\
\qquad\qquad\qquad\qquad\qquad\qquad -y(t)\mathrm{d}(N(t)),\quad t\geq 0,\\
y(t)=t+1,\quad t\in[-1,0].
\end{cases}
\tag{5.1}
$$

It is easy to verify that nonlinear neutral stochastic delay differential equation (5.1) satisfies the conditions of Theorem 3.1. The corresponding parameters are chosen as

$$
\lambda=\frac{1}{4},\quad \mu_1=5,\quad \mu_2=3,\quad K_1=64,\quad K_2=1,\quad K_3=1,\quad K_4=0.
$$

For $\theta\in[2-\sqrt{2},1)$, we take $\theta=\frac{3}{5}$ as an example. Then

$$
\begin{aligned}
\Delta t_0&=\min\left\{\frac{2(\mu_1-\mu_2)-|\kappa|(K_3+K_4)}{[(\theta-2)^2-2](K_1+K_2)+2|\kappa|^2(K_3+K_4)},1\right\}\\
&=\min\{0.65,1\}=0.65.
\end{aligned}
\tag{5.2}
$$

The initial condition is given by $y(t)=t+1, t\in[-1,0]$, where we take $\tau=1$. In the following tests, we show the influence of stepsize Δt and the parameter θ on M-S stability of the SST method, the data used in all figures are obtained by the mean square of data by 200 trajectories, that is $\mathrm{E}y_n^2 \approx \frac{1}{200}\sum_1^{200}[y_n^{(i)}]^2, \mathrm{E}y_n \approx \frac{1}{200}\sum_1^{200}[y_n^{(i)}]$, where $y_n^{(i)}$ denotes the numerical solution of y_n in the i th trajectory. Taking stepsizes $\Delta = 0.1,\ \Delta = 0.6$ and $\Delta = 0.8$, we obtain the

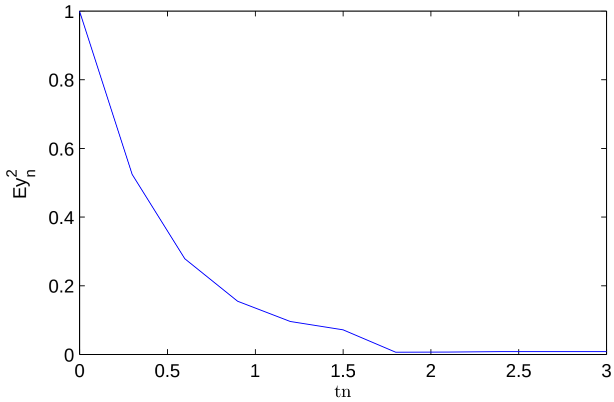

Fig. 5.1. Mean square stability of SST method with $\theta = 0.6$ and $\Delta t = 0.1$

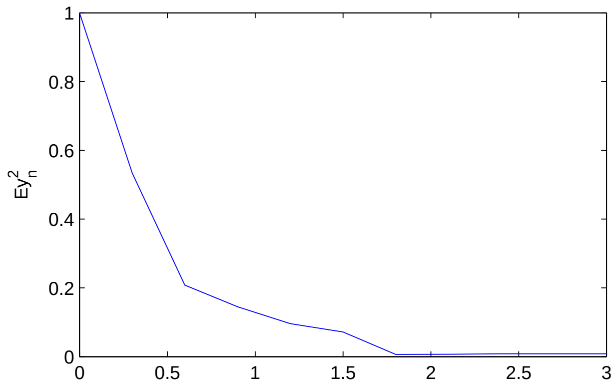

Fig. 5.2. Mean square stability of SST method with $\theta = 0.6$ and $\Delta t = 0.6$

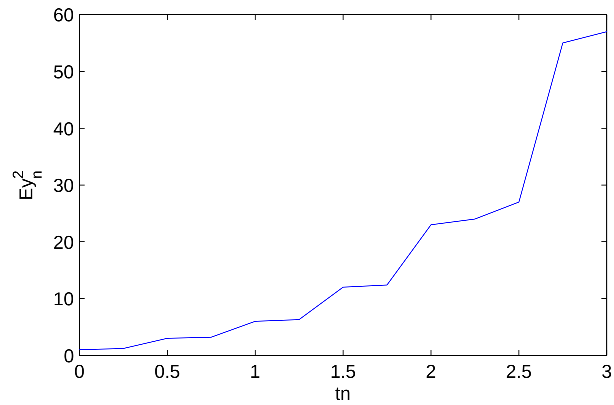

Fig. 5.3. Unstable test for SST method with $\theta = 0.6$ and $\Delta t = 0.8$

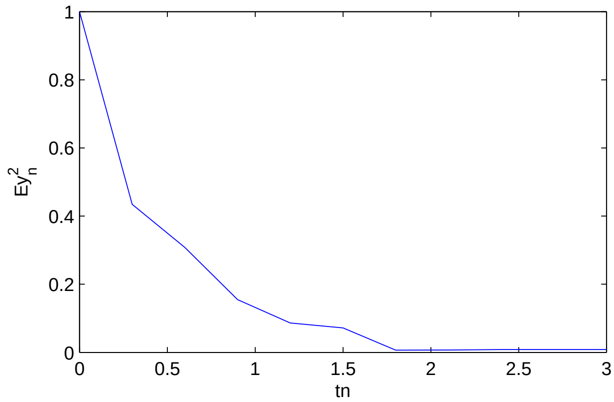

Fig. 5.4. Mean square stability of SST method with $\theta = 0.1$ and $\Delta t = 0.1$

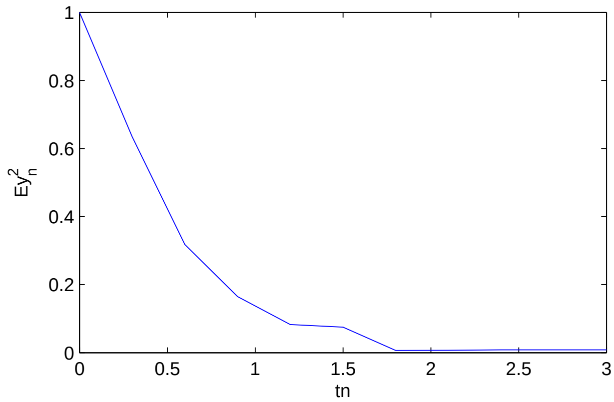

Fig. 5.5. Mean square stability of SST method with $\theta = 0.1$ and $\Delta t = 0.6$

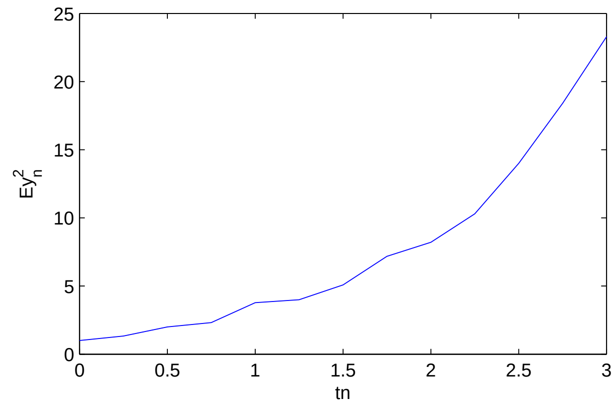

Fig. 5.6. Unstable test for SST method with $\theta = 0.1$ and $\Delta t = 0.8$

numerical solutions of (5.1). The numerical solutions are displayed in Figs 5.1-5.6 respectively.

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