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A NEW INTEGRAL EQUATION FORMULATION FOR SCATTERING BY PENETRABLE DIFFRACTION GRATINGS*

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Abstract

This paper is concerned with the problem of scattering of time-harmonic electromagnetic waves from penetrable diffraction gratings in the 2D polarization case. We propose a new, weakly singular, integral equation formulation for the scattering problem which is proved to be uniquely solvable. A main feature of the new integral equation formulation is that it avoids the computation of the normal derivative of double-layer potentials which is difficult and time consuming. A fast numerical algorithm is also developed for the scattering problem, based on the Nyström method for the new integral equation. Numerical examples are also shown to illustrate the applicability of the new integral equation formulation.

Mathematics subject classification: 74J20, 65R20.
Key words: Scattering problem, Transmission condition, Periodic interface, Diffraction gratings, Boundary integral equations, Helmholtz equation.

1. Introduction

The integral equation method plays an important role in both the theoretical analysis and the numerical solution of wave scattering problems (see, e.g., [10, 11, 20]). For example, the integral equation methods have been developed and studied for the scattering problems by bounded obstacles (sound-soft, sound-hard, and impedance cases) in [10, 11, 15], by periodic structures (called diffraction gratings in the physical and engineering sciences) in [2,17,18,22,23], and by unbounded rough surfaces in [6–8, 25, 26].

When the scattering obstacles or surfaces are penetrable, the scattering solution satisfies the transmission boundary conditions on the interfaces. In this case, the mostly used integral equations involve combined single- and double-layer boundary operators and their normal derivatives; see, e.g., [10, 12] for the bounded obstacle case and [23, 24] for the periodic surface case. In [19], several single integral equation formulations were developed for the transmission scattering problem, and in [9, 14], the multiple traces boundary integral equation formulations have been proposed for acoustic scattering by composite structures. It is remarked that spectral methods have also been developed in [13] for the periodic case, with the help of the form of

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quasi-periodic Green's functions. In all the above works, the integral equation formulations contain the hypersingular normal derivatives of double-layer boundary operators which are difficult and time consuming to be computed numerically.

Recently in [1], different from the classical integral equation approach for the scattering problem with a bounded penetrable obstacle, the scattered field is represented by a single-layer potential, leading to a new integral equation formulation which involves only single-layer boundary operators and their normal derivatives and therefore is weakly singular. The unique solvability of the new integral equation is established in [1] by employing the invertibility of the single-layer boundary operator.

Motivated by this, in this paper, we consider the problem of scattering of time-harmonic plane waves by a C^2-smooth periodic penetrable interface in $\mathbb{R}^2$. We use only single-layer potentials to represent the solution of the scattering problem, and then derive a new boundary integral equation formulation for the scattering problem by using the transmission boundary conditions on the interface. This new integral equation has a simpler form than the classical one and leads to a new, fast numerical method to solve the scattering problem. This numerical method avoids the computation of the normal derivatives of double-layer boundary operators, which occur in the classical boundary integral equation formulations and are difficult and time-consuming to be computed numerically. This new integral equation is proved to be equivalent to the scattering problem and also uniquely solvable. We will also show by numerical examples that the numerical algorithm based on this new integral equation is convergent.

This paper is organized as follows. In Section 2, we first introduce the scattering problem and then recall its existence and uniqueness results. We study some properties of the single- and double-layer boundary operators in periodic surfaces in Section 3 and review the classical boundary integral equations in Section 4. In Section 5, we present the new integral equation for the scattering problem and prove its equivalence to the scattering problem as well as its unique solvability. In Section 6, we discuss the Nyström method for the new integral equation in detail, and in Section 7, some numerical examples are given to show the convergence of the Nyström method. The final section 8 gives some conclusions.

2. The Scattering Problem

Suppose the penetrable diffraction grating Γ in $\mathbb{R}^2$ is given by

$$\Gamma = \{x \in \mathbb{R}^2 : x = (x_1, \gamma(x_1))\}$$

with a 2π-periodic function $\gamma \in C^2$. Then the whole space is divided by Γ into two parts:

$$\Omega_0 := \{x \in \mathbb{R}^2 \ : \ x_2 > \gamma(x_1)\} \quad \text{and} \quad \Omega_1 := \{x \in \mathbb{R}^2 \ : \ x_2 < \gamma(x_1)\}$$

which are filled with different homogeneous materials described by two different refractive indices $k_0^2 > 0$ and $k_1^2 > 0$, respectively.

Assume that a plane wave, which is given by

$$u^i = \exp(ik_0 x \cdot d) = \exp[ik_0(x_1 \cos\theta - x_2 \sin\theta)], \quad \theta \in (0, \pi),$$

is incident onto the penetrable grating from the top region Ω_0, where $d = (\cos\theta, -\sin\theta)$ is the incident direction, θ is the incident angle, $\alpha = k_0 \cos\theta$ and $\beta = k_0 \sin\theta$. Then the problem