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A COMPLETE CHARACTERIZATION OF THE ROBUST ISOLATED CALMNESS OF NUCLEAR NORM REGULARIZED CONVEX OPTIMIZATION PROBLEMS*

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Abstract

In this paper, we provide a complete characterization of the robust isolated calmness of the Karush-Kuhn-Tucker (KKT) solution mapping for convex constrained optimization problems regularized by the nuclear norm function. This study is motivated by the recent work in [8], where the authors show that under the Robinson constraint qualification at a local optimal solution, the KKT solution mapping for a wide class of conic programming problems is robustly isolated calm if and only if both the second order sufficient condition (SOSC) and the strict Robinson constraint qualification (SRCQ) are satisfied. Based on the variational properties of the nuclear norm function and its conjugate, we establish the equivalence between the primal/dual SOSC and the dual/primal SRCQ. The derived results lead to several equivalent characterizations of the robust isolated calmness of the KKT solution mapping and add insights to the existing literature on the stability of nuclear norm regularized convex optimization problems.

Mathematics subject classification: 90C25, 90C31, 65K10.
Key words: Robust isolated calmness, Nuclear norm, Second order sufficient condition, Strict Robinson constraint qualification

1. Introduction

Let $\mathbb{X}$ and $\mathbb{Y}$ be two finite dimensional Euclidean spaces. Let $G : \mathbb{X} \rightrightarrows \mathbb{Y}$ be a set-valued mapping. The graph of G is defined as $\operatorname{gph} G := \{(x, y) \in \mathbb{X} \times \mathbb{Y} \mid y \in G(x)\}$. Consider any $(\bar{x}, \bar{y}) \in \operatorname{gph} G$. The mapping G is said to be isolated calm at $\bar{x}$ for $\bar{y}$ if there exist a constant $\kappa > 0$ and open neighborhoods $\mathcal{X}$ of $\bar{x}$ and $\mathcal{Y}$ of $\bar{y}$ such that

$$G(x) \cap \mathcal{Y} \subset \{\bar{y}\} + \kappa \|x - \bar{x}\| \mathbb{B}_{\mathbb{Y}}, \quad \forall\, x \in \mathcal{X}, \tag{1.1}$$

where $\mathbb{B}_{\mathbb{Y}}$ is the unit ball in $\mathbb{Y}$ (cf. e.g., [9, 3.9 (3I)]). The mapping G is said to be robustly isolated calm at $\bar{x}$ for $\bar{y}$ if (1.1) holds and $G(x) \cap \mathcal{Y} \neq \emptyset$ for any $x \in \mathcal{X}$ [8, Definition 2].

In this paper, we are interested in characterizing the robust isolated calmness of the Karush-Kuhn-Tucker (KKT) solution mapping associated with the following nuclear norm regularized

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convex optimization problem:

$$\begin{array}{ll}\min\limits_{X} & h(\mathcal{F}X)+\langle C,X\rangle+\|X\|_* \\ \text{s.t.} & \mathcal{A}X-b\in\mathcal{Q},\end{array}\tag{1.2}$$

where the function $h:\mathbb{R}^d\to\mathbb{R}$ is twice continuously differentiable on $\operatorname{dom}h$, which is assumed to be a non-empty open convex set, and is also essentially strictly convex (i.e., h is strictly convex on every convex subset of $\operatorname{dom}\partial h$), $\mathcal{F}:\mathbb{R}^{m\times n}\to\mathbb{R}^d$ and $\mathcal{A}:\mathbb{R}^{m\times n}\to\mathbb{R}^e$ are linear operators, $C\in\mathbb{R}^{m\times n}$ and $b\in\mathbb{R}^e$ are given data, $\mathcal{Q}\subseteq\mathbb{R}^e$ is a nonempty convex polyhedral cone, $\|\cdot\|_*$ denotes the nuclear norm function in $\mathbb{R}^{m\times n}$, i.e., the sum of all the singular values of a given matrix, and m,n,d,e are non-negative integers. The nuclear norm regularizer has been extensively used in diverse disciplines due to its ability in promoting a low rank solution. See the references [2, 3, 14, 17, 20, 21] for a sample of applications.

The concept of the isolated calmness is of fundamental importance in variational analysis. The monograph [9] by Dontchev and Rockafellar contains a comprehensive study on this subject. Besides its own interest in sensitivity analysis and perturbation theory, the isolated calmness of the KKT solution mapping can be readily applied to provide linear rate convergence analysis for many important primal dual type methods such as the proximal augmented Lagrangian method [16] and the alternating direction method of multipliers [10] that can be employed efficiently to solve large scale matrix optimization problems such as (1.2). With this application in mind, in this paper we aim to derive easy-to-understand conditions to characterize the the isolated calmness of the KKT solution mapping for [10].

Obviously problem (1.2) can be equivalently formulated as the following conic programming problem

$$\begin{array}{ll}\min\limits_{X,t} & h(\mathcal{F}X)+\langle C,X\rangle+t \\ \text{s.t.} & \mathcal{A}X-b\in\mathcal{Q},\quad (X,t)\in\operatorname{epi}\|\cdot\|_*,\end{array}\tag{1.3}$$

where $\operatorname{epi}\|\cdot\|_*$ denotes the epigraph of the function $\|\cdot\|_*$. Since $\operatorname{epi}\|\cdot\|_*$ is not a polyhedral set, the sensitivity results in the conventional nonlinear programming are not applicable for problem (1.3). Recently, some progress has been achieved in characterizing the isolated calmness of KKT solution mappings for problems involving non-polyhedral functions. For example, Zhang and Zhang [23] show that for the nonlinear semidefinite programming, the second order sufficient condition (SOSC) and the strict Robinson constraint qualification (SRCQ) at a local optimal solution together are sufficient for the KKT solution mapping to be isolated calm. Adding to this result, Han et al. [11] show that the SRCQ is also necessary to ensure the isolated calmness of the KKT solution mapping for such problems. In [12], Liu and Pan extend the aforementioned results to problems constrained by the epigraph of the Ky Fan k-norm function. The most recent work of Ding et al. [8] indicates that under the Robinson constraint qualification (RCQ) at a local optimal solution, the KKT solution mapping for a wide class of conic programming is robustly isolated calm at the origin for a KKT point if and only if both the SOSC and the SRCQ hold at the reference point.

The results developed in [8] can be directly applied to problem (1.3). Thus, by examining the relationships between the SOSCs, the (strict) RCQs as well as the robust isolated calmness of the solution mappings corresponding to problem (1.2) and problem (1.3), we are able to extend the work in [8] to the nuclear norm regularized convex optimization problem (1.2). Additionally, due to the special structure of problem (1.2) and its dual, we could provide more insightful characterizations about the isolated calmness of the KKT solution mapping. Note