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BLOCK-CENTERED FINITE DIFFERENCE METHODS FOR NON-FICKIAN FLOW IN POROUS MEDIA*

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Abstract

In this article, two block-centered finite difference schemes are introduced and analyzed to solve the parabolic integro-differential equation arising in modeling non-Fickian flow in porous media. One scheme is Euler backward scheme with first order accuracy in time increment while the other is Crank-Nicolson scheme with second order accuracy in time increment. Stability analysis and second-order error estimates in spatial meshsize for both pressure and velocity in discrete L^2 norms are established on non-uniform rectangular grid. Numerical experiments using the schemes show that the convergence rates are in agreement with the theoretical analysis.

Mathematics subject classification: 65N06, 65N12, 65N15.
Key words: Block-centered finite difference, Parabolic integro-differential equation, Non-uniform, Error estimates, Numerical analysis.

1. Introduction

The non-Fickian flow in porous media is complicated by the history effect, which characterizes various mixing length growth of flow, which has been investigated, for example, in [1,2] and the references cited therein. This model of equation is very important in the transfer of reaction and passive contaminates in aquifers. This problem arises from many physical processes in which it is necessary to take into account the effects of memory due to the deficiency of the usual diffusion equations. It can serve as engineering model for nonlocal reactive transport in porous media [3,4]. It can also be used to model heat conduction with memory [3].

There are many papers on the numerical methods for this kind of problems. Finite volume methods for this problem were studied in [5, 6]. And finite element methods for this problem have been presented in [7]. In [8], some numerical methods for integro-differential equations of parabolic and hyperbolic types have been considered. And it is presented split least-squares finite element methods for non-fickian flow in porous media in [9]. Jiang [10] have considered mixed element methods for this problem when A, B are proportional to a unit matrix. And the L^2-error estimate and $L^\infty-$error estimate of mixed element methods for this problem in a general case are considered [1,2]. Besides, it is considered the backward euler mixed FEM and regularity of parabolic integrao-differential equations in [11].

In this paper we consider the block-centered finite difference methods for parabolic integro-differential equation arising in the modeling of non-Fickian flow in porous media. Finite difference method [17] is a very practical and primary method to solve the partial differential equation. In [12], a block-centered finite difference methods for the Darcy-Forchheimer model

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have been considered. Li and Rui present characteristic block-centred finite difference methods for nonlinear convection-dominated diffusion equation [13]. Besides, Liu and Li have applied the block-centered finite difference method to the nonlinear time-fractional parabolic equation in [14]. And in this paper, we present two block-centered finite difference schemes. One is Euler backward scheme with first order accuracy in time increment while the other is Crank-Nicolson scheme with second order accuracy in time increment. We also use some notations similar to [15]. We demonstrate that the proposed schemes are second-order error estimates in spatial meshsize for both pressure and velocity in discrete L^2 norms on non-uniform rectangular grid. Then we carry out some numerical examples to show the accuracy of the presented block-centered finite difference schemes. Compared with the other existing methods for the non-fickian flow in porous media, the applications of the block-centered finite difference methods enable us to approximate both the velocity and pressure with second-order accuracy, which is the superconvergence. Also the block-centered finite difference methods can guarantee local mass conservation. Besides, the applications of the block-centered finite difference methods enable us to transfer the saddle point problem to symmetric positive definite problem.

The paper is organized as follows. In Sect.2 we give the problem, some notations and lemmas. In Sect.3 we present the block-centered finite difference methods. In Sect.4 we present the stability analysis and error estimates for the presented methods. In Sect.5 some numerical experiments using the block-centered finite difference schemes are carried out. And the conclusion is given in Sect.6.

Through out the paper we use C, with or without subscript, to denote a positive constant, which could have different values at different appearances.

2. The Problem and Some Preliminaries

In this section, We consider the parabolic integro-differential equation arising in the modeling of non-Fickian flow in porous media. (see [1], [2]): find p=p(x, y, t) such that

$$\frac{\partial p}{\partial t} + \nabla \cdot \mathbf{u} = f(x,y,t), \qquad (x,y) \in \Omega, t \in J, \tag{2.1}$$

$$\mathbf{u} = -(\mathbf{A}\nabla p + \int_0^t \mathbf{B}(s)\nabla p(s)ds), \quad (x,y) \in \Omega, t \in J, \tag{2.2}$$

$$-\mathbf{A}\nabla p \cdot \mathbf{n} = 0, \qquad (x,y) \in \partial\Omega, t \in J, \tag{2.3}$$

$$p\big|_{t=0} = p_0(x,y), \qquad (x,y) \in \Omega. \tag{2.4}$$

Here $\Omega = (0,1) \times (0,1)$, $J = (0,1]$, $\mathbf{n}$ represents the unit exterior normal vector to the boundary of Ω,

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right), \quad \mathbf{A} = diag(a^x(x,y,t), a^y(x,y,t)), \quad \mathbf{B} = diag(b^x(x,y,t), b^y(x,y,t)).$$

We suppose that f, $\mathbf{A}$ and $\mathbf{B}$ are bounded smooth functions. And there exist positive constants $\alpha_1, \alpha_2, \theta_1, \theta_2$, such that

$$\alpha_1 \le a^x \le \alpha_2, \quad \alpha_1 \le a^y \le \alpha_2, \quad \theta_1 \le b^x \le \theta_2, \quad \theta_1 \le b^y \le \theta_2.$$

Also suppose that there exists a positive constant q, such that $|d_t a^{-l}| \le q, \ l = x, y.$