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A FULL DISCRETE STABILIZED METHOD FOR THE OPTIMAL CONTROL OF THE UNSTEADY NAVIER-STOKES EQUATIONS*

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Abstract

In this paper, a full discrete local projection stabilized (LPS) method is proposed to solve the optimal control problems of the unsteady Navier-Stokes equations with equal order elements. Convective effects and pressure are both stabilized by using the LPS method. A priori error estimates uniformly with respect to the Reynolds number are obtained, providing the true solutions are sufficient smooth. Numerical experiments are implemented to illustrate and confirm our theoretical analysis.

Mathematics subject classification: 49J20, 49K20, 65M12, 65M60.
Key words: Optimal control, Unsteady Navier-Stokes equations, High Reynolds number, Full discrete, Local projection stabilization.

1. Introduction

Optimal control problems of the unsteady Navier-Stokes equations (NSEs) derived from petroleum reservoir simulation, electrochemistry and other scientific or engineering applications. In applications, efficient and accurate numerical methods are always concerned.

Though there have been numerous studies on the finite element method (FEM) to solve NSEs, it is still a challenging work to use FEM to solve the NSEs. Since the standard Galerkin FEM may suffer from instability caused by convective effects and the lacking of inf-sup stability. The same difficulties also happen in the optimal control problems of the NSEs.

To overcome these two issues, some effective stabilization techniques are used, such as streamline upwind Petrov-Galerkin/pressure stabilized Petrov-Galerkin (SUPG/PSPG) methods [1–6], local projection stabilized (LPS) method [7–12], subgrid scale eddy viscosity method [13–17], and orthogonal subscales method [18, 19]. Among them, LPS method seems more attractive, since it does not need to calculate the higher order derivative and it is flexible in defining the projection in stabilized term. Gumermonds proposed LPS method to work on convection dominance problems [13]. The LPS method was used by Becker et al. [7] for stabilizing the pressure oscillation arising from violation of the deiscrte inf-sup condition.

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Lots of works ([8–12] for example) have been devoted to developing the LPS method for Oseen equations and stationary NSEs. Recently, Chen et al. [11] also analyzed the LPS method for the unsteady NSEs, and prove that the error estimates hold irrespective of the Reynolds number, providing the true solutions are sufficient smooth. Braack [20] adopted the LPS method to solve optimal control of NSEs equations.

In recent years, more and more researches are focus on the FMEs for the optimnal control problems of the NSEs or Oseen equations, see e.g., [21–26]. Among these reseraches, Chen et al. [27], Braack et al. [28], respectively, proposed subgrid scale eddy viscosity finite element method for optimal control of Oseen equations, which is a linear version of NSEs. As for the optimal control problems of the NSEs, we refer to Braack [20] and Yilmaz [29] for stabilization methods. In [20], a LPS scheme is presented without stability and error analysis. In [29], the error analysis is given for the semi-discrete scheme using inf-sup stable elements, and, the constant in the error analysis are dependent on the Reynolds number.

This paper continuous the research on the stabilization LPS method for the optimal control of the unsteady NSEs, based on the idea proposed in [11]. The optimal control problems are full discretized by using continuous equal-order finite elements in space and the reduced Crank-Nicolson scheme in time. A priori error estimates are obtained for the state, adjoint state and control variables by a special interpolation. The numerical experiments are shown, which illustrates and confirms our theoretical analysis. Compared with the work in [20] and the work in [29], the differences of this paper are as follows. First, a fully different full discrete scheme is adopted. Second, we focus on the equal-order velocity-pressure pairs, the stabilization method overcomes convection domination and circumvents the restrictive inf-sup condition. Third, the LPS method not only is a two-level approach but also is adaptive for pairs of space defined on the same mesh. Fourth, the error estimates are uniformly with Reynolds number, providing the true solutions are sufficient smooth.

The organization of the paper is as follows. In section 2, we present the optimal control problem. In section 3, we propose the full discrete local projection stabilized method for the optimal control problem. In section 4, we give error estimates. In section 5, we give numerical experiments. In Section 6, we conclude our study.

Throughout this paper, we use C to denote a positive constant independent of the $\triangle t$, h and ν, not necessarily the same at each occurrence.

2. The Optimal Control Problem

Let $\Omega \subset \mathbb{R}^d$ and $\Omega_U \subset \mathbb{R}^d$ $(d = 2, 3)$ be two bounded domains with polygonal or polyhedral boundary $\Gamma = \partial\Omega$ and $\Gamma_U = \partial\Omega_U$, respectively. Let $Q = [0, T] \times \Omega$, where $T > 0$. We consider the following optimal control problem to minimize

$$J(\boldsymbol{y}, \boldsymbol{u}) = \frac{1}{2}\int_Q ((\boldsymbol{y}(\boldsymbol{x}, t) - \boldsymbol{y}_d(\boldsymbol{x}, t))^2 + \alpha(\boldsymbol{u}(\boldsymbol{x}, t))^2) d\boldsymbol{x}dt. \tag{2.1}$$

subject to the unsteady Naiver-Stokes equations

$$\begin{cases} \boldsymbol{y}_t - \nu\triangle\boldsymbol{y} + (\boldsymbol{y}\cdot\nabla)\boldsymbol{y} + \nabla r = \boldsymbol{f} + B\boldsymbol{u}, & \text{in } Q, \\ \nabla\cdot\boldsymbol{y} = 0, & \text{in } Q, \\ \boldsymbol{y} = \boldsymbol{0}, & \text{on } [0, T]\times\Gamma, \\ \boldsymbol{y}(\boldsymbol{x}, 0) = \boldsymbol{y}_0(\boldsymbol{x}), & \text{in } \Omega, \end{cases} \tag{2.2}$$