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A TRUST-REGION ALGORITHM FOR SOLVING MINI-MAX PROBLEM*

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Abstract

In this paper, we propose an algorithm for solving inequality constrained mini-max optimization problem. In this algorithm, an active set strategy is used together with multiplier method to convert the inequality constrained mini-max optimization problem into unconstrained optimization problem. A trust-region method is a well-accepted technique in constrained optimization to assure global convergence and is more robust when they deal with rounding errors. One of the advantages of trust-region method is that it does not require the objective function of the model to be convex.

A global convergence analysis for the proposed algorithm is presented under some conditions. To show the efficiency of the algorithm numerical results for a number of test problems are reported.

Mathematics subject classification: 90C30, 90B50, 65K05, 62C20.
Key words: Mini-max problem, Active-set, Multiplier method, Trust-region, Global convergence.

1. Introduction

Many real world applications can be modeled as a mini-max optimization problem. This problem arises in engineering design, computer-aided design, circuit design, chemical design, systems of nonlinear equations, problems of finding feasible points of systems of inequalities, nonlinear programming problems, multi objective problems, optimal control and others. Theoretical study for the mini-max optimization problem can be found in [1, 2].

In this paper, we introduce an active-set trust-region algorithm to solve the following mini-max problem

$$\begin{array}{ll} \min_{x\in\Re^n} & \Psi(x), \\ \text{subject } to & h(x) \leq 0, \end{array} \tag{1.1}$$

where $\Psi(x) = \max_{1\leq i\leq m} f_i(x)$. The functions $f_i : \Re^n \to \Re$, $i = 1, ..., m$, and $h(x) : \Re^n \to \Re^p$, are twice continuously differentiable. The objective function $\Psi(x)$ is not necessarily differentiable even though the functions $f_i(x)$, $i = 1, ..., m$, are all differentiable. So, the classical algorithms which are using for solving smooth nonlinear programming problems can not be applied directly on Problem (1.1). There are several types of algorithms suggested to solve min-max problems, see [3–13]. The first type of algorithms shows the Problem (1.1) as a constrained non-smooth optimization problem. Therefore, general methods is used to solve it, see [14, 15]. The second type of algorithms solves the Problem (1.1) by considering the special structure of its non-differentiability so as to make use of certain smooth optimization methods, see [4, 16]. The third type of algorithms solves the Problem (1.1) by converting it into

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an equivalent smooth inequality constrained optimization problem by inserting a new variable $z \in \Re$.

$$\begin{array}{ll} \min_{(x^T,z)} & z \\ \textit{subject to} & h(x) \le 0, \\ & f_i(x) - z \le 0, \qquad i = 1, ..., m. \end{array}$$

It is obviously implies that solving the finite min-max inequality constrained Problem (1.1) is equivalent to solve the above problem, see [1,2]. In this paper, the proposed approach belongs to the third type.

The above problem can be summarized as follows

$$\begin{array}{ll} \min_{\tilde{x}} & F(\tilde{x}) \\ \textit{subject to} & G(\tilde{x}) \le 0, \end{array} \tag{1.2}$$

where $\tilde{x}$ represent the vector $(x^T, z) \in \Re^{n+1}$, $F(\tilde{x}) = z$, and $G(\tilde{x}) \in \Re^{m+p}$ is a vector whose elements are $(h(x), f_i(x) - z)^T$, $i = 1, ..., m$.

The Lagrangian function associated with Problem (1.2) is the function

$$\ell(\tilde{x}, \lambda) = F(\tilde{x}) + \lambda^T G(\tilde{x}), \tag{1.3}$$

where $\lambda \in \Re^{m+p}$ is the Lagrange multiplier vector associated with inequality constraints $G(\tilde{x})$. Let $J(\tilde{x})$ be the set of indices of violated or binding inequality constraints at a point x. i.e., $J(\tilde{x}) = \{j : G_j(\tilde{x}) \ge 0\}$. If the vectors in the set $\{\nabla G_j(\tilde{x}), j \in J(\tilde{x}_*)\}$ are linearly independent, then the point $\tilde{x}_*$ is called a regular point for Problem (1.2).

The first-order necessary conditions for the regular point $\tilde{x}_*$ to be a local minimizer of Problem (1.2) are the existence of the multiplier vector $\lambda_* \in \Re^{m+p}$ such that $(\tilde{x}_*, \lambda_*)$ satisfies

$$\nabla_{\tilde{x}} F(\tilde{x}_*) + \nabla_{\tilde{x}} G(\tilde{x}_*)\lambda_* = 0, \tag{1.4}$$

$$G(\tilde{x}_*) \le 0, \tag{1.5}$$

$$(\lambda_*)_i G_i(\tilde{x_*}) = 0, \qquad i = 1, ..., m+p, \tag{1.6}$$

$$(\lambda_*)_i \ge 0, \qquad i = 1, ..., m+p. \tag{1.7}$$

Conditions (1.4)-(1.7) are also known as the Karush-Kuhn-Tucker conditions or the KKT conditions. A point $(\tilde{x}_*, \lambda_*)$ that satisfies the KKT conditions is called a KKT point. For more details, see [17].

In this paper an active set strategy is used together with a multiplier method to convert Problem (1.2) into unconstrained optimization problem. The general idea behind the active-set strategy is to identify at every iteration, the active inequality constraints and treat them as equalities. This allows the use of the well-developed techniques for solving the equality constrained optimization problems. Many authors have proposed active-set algorithms for solving general nonlinear programming problems, see, e.g., [18–21].

Following the active set strategy in [18], we define a 0-1 diagonal indicator matrix $D(x) \in \Re^{m+p \times m+p}$, whose diagonal entries are

$$d_i(\tilde{x}) = \begin{cases} 1 & \text{if } G_i(\tilde{x}) \ge 0, \\ 0 & \text{if } G_i(\tilde{x}) < 0. \end{cases} \tag{1.8}$$

Using the above matrix, Problem (1.2) is converted to the following problem

$$\begin{array}{ll} \min & F(\tilde{x}), \\ \textit{subject to} & G(\tilde{x})^T D(\tilde{x}) G(\tilde{x}) = 0. \end{array}$$